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Basic Computational Skills and Calculation Strategies among High-Achieving Secondary School Students in South Africa: An Investigation of Their Effects on Student Performance

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Abstract

Purpose: This study aims to comprehensively assess the basic computational skills and calculation strategies of high-achieving Grade 10 students in South Africa and to examine how these skills influence their performance in operations involving decimals and fractions. In particular, it explores whether persistent reliance on counting-based procedures limits students' conceptual understanding of place value and constrains their ability to flexibly apply more advanced numerical strategies. By focusing on high-achieving students, the study seeks to clarify whether foundational weaknesses remain even among those considered academically successful.

Methodology: A paper-based computational assessment was administered to 30 high-achieving Grade 10 students in a public secondary school in Mpumalanga Province. The test covered the four operations with integers, as well as operations involving decimals and fractions. Calculators were not permitted to ensure that students' mental and written strategies could be observed. Descriptive statistics were calculated to determine overall performance patterns, and detailed error analyses were conducted to identify strategy use, recurring mistakes, and underlying misconceptions related to place value, equivalence, and multiplicative reasoning.

Findings: The overall mean correct response rate was 58.4%. Students performed relatively well in addition, subtraction, and basic single-digit multiplication, but achievement declined substantially in multi-digit multiplication and division. Even lower performance was observed in decimal operations and particularly in fraction tasks. Across integer items, students relied predominantly on counting strategies, while more efficient approaches such as retrieval, decomposition, or structured algorithms were rarely used. Frequent errors included counting inaccuracies, weak place-value coordination, incorrect alignment in written algorithms, and misunderstandings of decimal shifts. In fraction tasks, both conceptual and procedural errors were evident, indicating fragile understanding of fundamental fraction concepts.

Unique Contribution to Theory, Practice and Policy: The study contributes to theory by demonstrating how sustained dependence on counting may inhibit the development of place-value understanding and higher-order numerical reasoning. For practice, it highlights the importance of explicitly promoting efficient calculation strategies alongside conceptual instruction in the base-ten system. At the policy level, the findings suggest the need to strengthen early mathematics curricula and instructional support systems to prevent the persistence of foundational computational weaknesses into secondary education.

Keywords: *Computational Skills, Counting Strategies, Place Value Understanding, Misconceptions, Mathematical Equivalence*

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INTRODUCTION

The Republic of South Africa (hereafter, South Africa) has participated in the Trends in International Mathematics and Science Study (TIMSS) since 1995; however, compared with other participating countries, its levels of mathematics achievement and educational outcomes still remain low (Mokgwathi et al., 2019; Reddy et al., 2020; Reddy et al., 2022). Although statistically significant increases were observed between 2003 and 2019, the rate of increase has declined, and the results have not yet fully aligned with the country's development goals (Reddy et al., 2022). In the 2023 survey, South Africa ranked 58th out of 58 participating countries at the Grade 4 level and 38th out of 44 countries at the Grade 8 level, indicating extremely low performance (TIMSS & PIRLS International Study Center, 2024a; 2024b). In addition, in the mathematics component of the Annual National Assessment conducted in 2014, the average score of Grade 9 students was 10.8 out of 100, and the average score of Grade 6 students was below 50%. These results suggest that problems already clearly exist in the arithmetic content of primary education, even before the domains addressed in lower secondary education (Department of Basic Education, 2014). In particular, insufficient conceptual understanding of basic arithmetic operations is considered to have become clearly evident. In South Africa, when performing the four basic arithmetic operations, a method is often used in which learners draw multiple tally marks and count them to obtain an answer. As a result, numbers for learners function not as integers but merely as labels or positions (Japan International Cooperation Agency, 2024). This strategy is commonly referred to as counting, and reliance on counting may not lead to a conceptual understanding of place value and may limit the development of higher-order computational abilities (Uchida, 2009). In addition, conceptual understanding of place value and the coordination and integration of units have been shown to influence not only integer multiplication but also knowledge of fractions (Hackenberg & Sevinc, 2024). Furthermore, counting in groups and recognizing numerical structures constitute mathematical patterning and structural thinking, which have been suggested to help children later abstract and generalize mathematical ideas and relationships (Mulligan et al., 2015). Among learners in South Africa, insufficient understanding of decimals and fractions is particularly serious, and reliance on counting may be one of the contributing factors (Japan International Cooperation Agency, 2024).

Based on the above, it can be considered that children in South Africa, by relying on counting from the primary education stage, may fail to develop a conceptual understanding of place value, resulting in limitations in the development of higher-order computational abilities and a weak bridge to abstraction and generalization. On the other hand, the actual situation of students in secondary education has remained largely unclear. Therefore, this study aims to clarify secondary school students' computational ability related to the four arithmetic operations with integers (hereafter, basic computational skills) and their calculation strategies, and to analyze and discuss the influence of these factors on the learning of computations beyond the four arithmetic operations with integers.

LITERATURE REVIEW

In this chapter, previous studies related to the present study are reviewed by categorizing them into the domains of addition, multiplication tables, decimals, and fractions. Through this review, the main characteristics of strategies and error patterns in each domain are identified, and the positioning of the purpose and research questions of the present study is further clarified.

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Addition Strategies and Strategy Selection

Children generally use three types of addition strategies: counting, retrieval, and decomposition (Geary et al., 1992; Geary et al., 1996; Shrager & Siegler, 1998). Counting is among the first to emerge in children's repertoire of addition strategies. To begin with, counting strategies involve enumerating both of the addends, "count-all." During the preschool years, children acquire a more advanced counting strategy "count-on," which involves counting up from one addend the value of the second addend (Siegler & Robinson, 1982). The retrieval strategy involves recalling the solution to a problem as a number fact stored in memory, rather than active computation. Decomposition involves transforming the original problem into two or more simpler problems, using either a previously memorized number fact or the base-10 properties of the number system (Laski et al., 2014).

Children generally rely primarily on counting strategies at an early age; however, they are capable of using multiple strategies. The frequency with which children use particular strategies is related to problem difficulty and prior knowledge (Geary et al., 2004; Kerkman & Siegler, 1993; Laski et al., 2013; Lemaire Callies, 2009; Siegler, 1988). As problem difficulty increases, children tend to use more advanced strategies in order to maximize efficiency while maintaining accuracy (Laski et al., 2014).

Single-digit Multiplication Strategies and Strategy Selection

Children use several types of strategies to perform single-digit multiplication, including the count-all strategy, additive calculation strategy, count-by strategy, multiplication triads, and hybrid strategies (Sherin & Fuson, 2005). The count-all strategy involves counting all numbers from 1 to the product, and students use this strategy in various ways, such as drawing pictures and counting them. The additive calculation strategy involves repeatedly adding the multiplier or multiplicand and is more efficient than the count-all strategy (Sherin & Fuson, 2005). The count-by strategy involves treating learned multiples as a numerical sequence and using a memorized string to produce the answer (Sherin & Fuson, 2005). It is the strategy in which errors are most likely to occur (Steel & Funnell, 2001). The multiplication triads involve utilizing the relational structure among a set of three numbers. The hybrid strategies combine multiple strategies; for example, the student first uses the multiplication triads and then applies the additive calculation strategy (Sherin & Fuson, 2005). The pattern-based strategies involve the use of numerical regularities inherent to numbers, such as general rules including $N \times 0 = 0$, $N \times 1 = N$, and $N \times 10 = N0$, as well as patterns specific to the number 9. In such cases, many students respond immediately without engaging in any visible computational procedures (Sherin & Fuson, 2005).

Several structural features that are thought to play an important role in students' solution methods for single-digit multiplication have been identified (Sherin & Fuson, 2005). For example, students are more likely to use the multiplication triads when the multiplicand is small, when either the multiplier or multiplicand is 5, or when the multiplier and multiplicand are the same number (Campbell & Graham, 1985; LeFevre et al., 1996; Miller et al., 1984). In addition, a study of adults by LeFevre et al. (1996) showed that the additive calculation strategy was primarily used for problems in which the multiplier or multiplicand was 2, the count-by strategy was primarily used for problems in which the multiplier or multiplicand was 3 or 5, and the multiplication triads was primarily used for problems in which the product exceeded 40. For problems in which the multiplier or multiplicand was 0 or 1, only the multiplication triads or pattern-based strategies methods were used.

Errors in Decimal Arithmetic and Their Causes

According to a study of lower secondary school students conducted by Standiford (1982), errors observed in decimal addition and subtraction excluding those attributable to errors in integer addition and subtraction can be classified into seven main types. The first type involves the misapplication of multiplication rules to determine the decimal places. Students who make this error may have misconceptions about decimals and may be unable to distinguish between the treatment of decimals in addition and multiplication (Ojose, 2015). The second type involves errors resulting from the application of a right justification, in which students write numbers so that the digits are aligned at the right end when performing calculations with column method. In this case, students may place the decimal point in a position corresponding to one addend, place it in both positions, apply multiplication rules to determine its placement, or omit it altogether. The third type involves errors in which students write numbers without explicit decimal points so that the digits are aligned at the right end when performing calculations with column method. The fourth type involves errors resulting from the application of a left justification, in which students write numbers so that the digits are aligned at the left end when performing calculations with column method. The placement of the decimal point in this case is similar to that observed under the right justification. The fifth type involves errors in which zeros written between the decimal point and the first non-zero digit to its right are omitted. Lai & Tsang (2009) showed that students who make errors of the second to fifth types have limited conceptual knowledge regarding the relationship between the decimal number system and the rational number system. The sixth type involves errors in which multiple decimal points appear, and the seventh type involves errors in which students omit a required decimal point. In addition, in addition problems only, there are errors in which the decimal part expands when a carry from the decimal place to the ones place is required (Standiford, 1982). In subtraction problems only, there are also errors in which students incorrectly assume that the longer number is the larger number and therefore always treat it as the minuend and align digits to the right when calculating, as well as errors in which students simply bring down the digits of the subtrahend when there is no corresponding digit in the minuend (Standiford, 1982). The error of assuming that the longer number is the larger number is mainly attributable to students' incomplete understanding of place value and the fundamental concept of decimals (Lai et al., 2009).

According to a study of undergraduate education majors in the United States by Joung & Kim (2022), errors observed in decimal multiplication and division—excluding those attributable to errors in integer multiplication and division—can be classified into two types. The first type is misunderstanding of the place value. This type involves errors in which the decimal point is placed incorrectly. Students who make these errors may have limited knowledge of decimal place values and a limited conceptual understanding of place value (Joung & Kim, 2022). The second type is misunderstanding of mathematical equivalence. This type involves errors resulting from a lack of knowledge of the definition of mathematical equivalence. Students who make these errors may have insufficient knowledge regarding mathematical equivalence (Joung & Kim, 2022). In addition, in division problems only, there is a misconception in which students reverse the positions of the divisor and the dividend. This error is caused by an insufficient understanding of place value and suggests that students have limited knowledge regarding the definition of division and the definition of the decimals (Lai & Murray, 2014).

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Errors in Fraction Arithmetic and Their Causes

Mutiara (2025) identified the types and causes of errors made by students when solving fraction addition and subtraction problems through a study involving 32 fourth-grade students in Indonesia. This study was conducted using a descriptive survey with the aim of identifying the types and causes of errors in the four arithmetic operations with fractions among fourth-grade elementary school students and obtaining implications for improving instruction. The types and causes of incorrect responses were classified into the following four categories. The first type is conceptual errors. These are errors that occur when students pret mathematical concepts. For example, students may multiply numerators and denominators respectively, calculate numerators and denominators separately without finding a common denominator, add numerators and multiply denominators, add the numerators after finding a common denominator but write only the resulting integer as the answer, or subtract the numerators while adding the denominators. The cause of these errors is considered to be students' lack of understanding or inability to interpret the fundamental concepts of fractions and fraction addition and subtraction. The second type is principle errors. These are errors that occur when students attempt to relate multiple facts or some concepts. The cause of these errors is insufficient acquisition of knowledge and prerequisite skills. The third type is operation errors. These are errors that occur in computation, algebra, or other mathematical tasks. The cause of these errors lies in forgetting to count, insufficient mastery of multiplication, or failure to pay attention during calculation. The fourth type is mistakes due to carelessness or lack of thoroughness. For example, students may misread operation symbols, make transcription errors, or fail to write the final answers. The cause of these errors is inattention and rushing through problem solving.

Lin et al. (2025) conducted a meta-analysis of 17 studies on error patterns among students with difficulties in mathematics and identified types of errors that commonly appear across the mathematical domains examined. The total number of participants was 1,122, with most being elementary and lower secondary school students, and a smaller number from high school and university. In this analysis, Lin et al. (2025) identified two types of incorrect responses in fraction multiplication and division problems. The first type involves errors resulting from the incorrect application of addition rules to multiplication and division. These errors are caused by students memorizing procedures without understanding algorithms or rules. In addition, students who make these errors are suggested to have difficulty determining a common denominator and weak conceptual knowledge of equivalent fractions (Moyo & Machaba, 2022). The second type involves errors resulting from failing to convert the divisor, which occurs only in division. Students perform multiplication without taking the reciprocal. The cause of this error, similar to the cause of incorrectly applying addition rules to multiplication and division, is considered to be students' insufficient understanding of the fundamental concepts of fractions and learning multiplication and division before adequately mastering addition skills (Lin et al., 2025). In addition, there are errors similar to cross multiplication, in which students multiply the numerator of the first fraction by the denominator of the second fraction and multiply the denominator of the first fraction by the numerator of the second fraction. Persisting in such an error can hinder rational reasoning and lead to the integration of incorrect concepts (Ojose, 2015).

Purpose and Research Questions

As noted previously, although the challenges faced by students in South Africa have been clarified in prior research, empirical studies conducted in South Africa remain limited. In particular, there are no studies that collectively assess difficulties related to the four arithmetic operations involving integers, decimals, and fractions in order to comprehensively understand students' actual computational performance. Moreover, the actual situation of students in upper secondary education in South Africa, as well as that of high-achieving students, has not been clarified. Clarifying these issues is expected to enable the identification of domains in which students experience difficulties and to obtain implications for improving mathematics education.

Based on the above considerations, the purpose of this study is to comprehensively assess and analyze the basic computational skills, as well as the calculation strategies and difficulties related to the four arithmetic operations involving decimals and fractions, of a convenience sample of high-achieving students enrolled in upper secondary schools in South Africa. Furthermore, for the target students, the following two points were set as the research questions.

i. What level of basic computational skills do the students possess, and what types of calculation strategies do they use? What types and causes of incorrect answers are observed, and what difficulties do they face?
ii. To what extent are the students able to solve problems involving the four arithmetic operations with decimals and fractions? What types and causes of incorrect answers are observed, and what difficulties do they face?

METHODOLOGY

Participants

On February 15 and 22, 2023, a paper-based assessment was conducted. The participants were 30 Grade 10 students enrolled in a public upper secondary school in Mpumalanga Province, South Africa, representing approximately the top 10% of academic performers. They were selected based on their results in the school's internal examinations from the previous academic year. The academic achievement level of this school is considered to be average compared with other schools in the same region. Participants were informed in advance of the research purpose, content, duration, voluntary nature of participation, and their right to withdraw at any time, and informed consent was obtained. Participation or non-participation had no effect at all on grades or school evaluations. No personally identifiable information was collected, and all data were anonymized and securely managed. The study was conducted outside regular class time with the permission of the school principal and the responsible teacher, and care was taken to minimize psychological burden.

Test Instruments

A computational test was developed and administered. The test consisted of problems involving addition of natural numbers (hereafter, addition), subtraction of natural numbers (hereafter, subtraction), multiplication of natural numbers in which both the multiplicand and the multiplier were limited to numbers less than or equal to 10 (hereafter, basic multiplication), multiplication of natural numbers in which either the multiplicand or the multiplier was a natural number with two or more digits (hereafter, multiplication), division of natural numbers (hereafter, division), the four arithmetic operations with decimals (hereafter, decimals), and the four arithmetic operations with fractions (hereafter, fractions). Sufficient time was allocated to allow students to review and attempt all items. The use of calculators was not permitted under any circumstances. In this study, answers were considered correct even if fractions were not expressed in their reduced form.

Data Analysis

Based on the test results, descriptive statistics were calculated for each survey item and for the total score. Furthermore, the correct response rate for each problem and the types of incorrect responses were organized, and the causes of the incorrect responses were analyzed from the perspectives of calculation strategies and conceptual understanding, based on the previous studies described above.

RESULTS AND DISCUSSION

Descriptive Statistics

The correct response rates for each item as well as, the mean, maximum, minimum, standard deviation, kurtosis, and skewness of the correct response rates for each computational content area and for the overall test are summarized in Tables 1 and 2, respectively. The mean overall correct response rate was 58.4%, indicating that the results were not satisfactory. The correct response rates for addition, subtraction, and basic multiplication were relatively high, whereas those for multiplication, division, decimals, and fractions were low, with fractions showing the lowest correct response rate. Based on the standard deviation, skewness, and kurtosis, substantial individual differences were observed for subtraction, multiplication, and division.

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Table 1: Correct Response Rates by Item

Item	Description	CR rates %
1) 3 + 4	One-digit + one-digit	100
2) 23 + 6	Two-digit + one-digit (no carrying)	100
3) 49 + 8	Two-digit + one-digit (with carrying)	83
4) 65 + 17	Two-digit + two-digit (with carrying)	70
5) 757 + 87	Three-digit + two-digit (with carrying)	43
6) 8 - 5	One-digit − one-digit	100
7) 28 - 4	Two-digit − one-digit (no borrowing)	100
8) 81 - 8	Two-digit − one-digit (with borrowing)	70
9) 72 - 57	Two-digit − two-digit (with borrowing)	50
10) 440 - 67	Three-digit − two-digit (with borrowing)	23
11) 1 × 3	Multiplication by 1	100
12) 2 × 7	Multiplication by 2	100
13) 5 × 6	Multiplication by 5	100
14) 3 × 8	Multiplication by 3	73
15) 4 × 7	Multiplication by 4	77
16) 6 × 9	Multiplication by 6	23
17) 7 × 6	Multiplication by 7	30
18) 8 × 4	Multiplication by 8	40
19) 9 × 9	Multiplication by 9	60
20) 10 × 7	Multiplication by 10	93
21) 30 × 20	Multiplication of multiples of 10	57
22) 40 × 800	Multiplication of multiples of 100	30
23) 32 × 3	Two-digit × one-digit (no carrying)	77
24) 93 × 4	Two-digit × one-digit (with carrying)	47
25) 38 × 67	Two-digit × two-digit	37
26) 24 ÷ 6	Two-digit ÷ one-digit (no remainder)	77
27) 306 ÷ 2	Three-digit ÷ one-digit (no remainder)	70
28) 589 ÷ 31	Three-digit ÷ two-digit (no remainder)	33
29) 6 + 2.5	Integer + decimal (no carrying)	87
30) 8.4 + 7.5	Decimal + decimal (no carrying)	90
31) 6.6 - 4	Decimal − integer (no borrowing)	80
32) 3.8 - 2.4	Decimal − decimal (no borrowing)	93
33) 1.2 × 3	Decimal × integer (no carrying)	60
34) 4.3 × 0.2	Decimal × decimal (no carrying)	3
35) 35 ÷ 0.5	Integer ÷ decimal	20
36) 1.2 ÷ 4	Decimal ÷ integer	23
37) $\frac{5}{9}+\frac{2}{9}$	Fraction addition (same denominator)	97
38) $\frac{8}{9}-\frac{2}{9}$	Fraction subtraction (same denominator)	90
39) $\frac{1}{4}+\frac{2}{2}$	Fraction addition (denominators in a multiple relationship)	30
40) $\frac{1}{2}-\frac{2}{5}$	Fraction subtraction (coprime denominators)	40
41) $\frac{3}{4}+\frac{1}{6}$	Fraction addition (denominators with a common factor)	27
42) $\frac{3}{4}\times\frac{20}{9}$	Fraction × fraction	50
43) $\frac{5}{2}\times 0.3$	Fraction × decimal	13
44) $\frac{3}{8}\div 6$	Fraction ÷ integer	13
45) $\frac{8}{5}\div\frac{4}{9}$	Fraction ÷ fraction	13
46) $\frac{9}{8}\div 0.36$	Fraction ÷ decimal	0

Note. CR represents correct response.

Table 2: Descriptive Statistics of Correct Response Rates by Computational Content

Item	M	SD	Min	Max	Skewness	Kurtosis
Addition	79.3	17.0	40.0	100.0	-0.3	-0.7
Subtraction	68.0	20.7	40.0	100.0	0.1	-1.1
Basic multiplication	70.0	14.1	40.0	100.0	0.2	-0.3
Multiplication	48.0	29.1	0.0	100.0	-0.1	-0.7
Division	60.0	29.6	0.0	100.0	0.1	-1.1
Decimals	57.1	17.9	13.0	100.0	0.2	0.8
Fractions	37.3	17.0	10.0	80.0	0.8	0.0
Overall	**58.4**	**12.7**	**37.0**	**84.8**	**0.3**	**-0.6**

Note: M, SD, Min, and Max represent standard mean, deviation, minimum value, and maximum value, respectively.

Error Analysis

For Items 1) to 10), all students relied on finger-based counting strategies, which is consistent with previous studies (Japan International Cooperation Agency, 2024). Excluding non-responses, the causes of incorrect answers were due to counting errors. For example, although students used column method for multi-digit calculations, incorrect answers resulted from counting mistakes.

For Items 11) to 20), errors such as answering 6 × 9 as 52, 56, or 48, and answering 7 × 6 as 43 or 36 were observed. These incorrect answers involved errors of ±1 or ±2, as well as errors caused by over-addition or under-addition. Although some use of strategies such as pattern-based strategies, additive calculation strategy, and commutative property was observed, the continued reliance on counting strategies in all cases was identified as the underlying cause. These incorrect answers are consistent with those observed among Grade 2 students in Zambia, who demonstrated insufficient understanding of numbers as composite units and of repeated addition (Nakawa, 2016). Given that the strategies selected were also the same, it is suggested that students in South Africa may face similar difficulties.

For Items 21) and 25), students solved the problems using column method, additive calculation strategy, and the lattice method; however, in all cases, they continued to rely on counting strategies, and incorrect answers due to counting errors were consistently observed. For example, incorrect answers such as 40 × 800 = 28,000 or 36,000 were noted. In addition, answers in which calculations were left incomplete due to slow counting speed were also observed, particularly in Item 25), where 10 such cases were identified. Furthermore, nine instances of incorrect answers such as 20 × 30 = 60 and 40 × 800 = 3,200 were observed. These errors suggest that students may have limitations in their understanding of place value (Uchida, 2009).

For Items 26) and 28), students used column method and additive calculation strategy to solve the problems. For example, in Item 28), 17 responses—the most frequently observed—used a method of repeatedly adding 31, 93, three sets of 31, or 124, four sets of 31. The strategy of drawing tally marks and counting them, as reported in previous studies (Japan International Cooperation Agency, 2024), was also observed. In all cases, reliance on counting strategies persisted, and incorrect answers due to counting errors continued to be observed. In addition, incorrect answers such as 306 ÷ 2 = 19 or 23 and 589 ÷ 31 = 109 were observed. These results suggest that students' number sense may be limited to ordinal understanding or counting, which is consistent with previous studies (Japan International Cooperation Agency, 2024).

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For Items 29) to 36), students continued to use counting strategies and additive calculation strategy; however, incorrect answers due to counting errors became relatively less frequent. On the other hand, almost all incorrect answers were consistent with those reported in previous studies (Tables 3 and 4). In Item 34), incorrect answers such as 4.3 × 0.2 = 4.6 and 4.3 × 0.2 = 0.6, which reflect misunderstanding regarding both place value and mathematical equivalence, were observed in 18 students. These findings indicate that more than half of the students may have limited knowledge of decimal place value and limited conceptual understanding of place value, as well as insufficient knowledge of mathematical equivalence. Only one student answered both Items 33) and 34) correctly, indicating that all other students experienced at least one of these difficulties. Supporting this result, incorrect answers due to misconceptions about mathematical equivalence and misunderstandings of place value were also observed in Items 35) and 36).

Table 3: Error Types, Rates, and Frequencies for Items 29 to 32

Error types	Examples	Rates %	Frequencies
Misapplication of multiplication rules (Standiford, 1982)	6 + 2.5 = 3.1, 6.6 – 4 = 6.2	20.0	7
Missing decimal point (Standiford, 1982)	3.8 – 2.4 = 14	3.3	1
Decimal part expanding (Standiford, 1982)	6 + 2.5 = 2.11	3.3	1
Careless addition error	3.8 – 2.4 = 6.2	3.3	1
Careless subtraction error	8.4 + 7.5 = 0.9	3.3	1
Counting mistakes	8.4 + 7.5 = 15.8	6.7	2

Note. Rates and Frequencies indicate the proportion of students who made the error and the number of occurrences of that error respectively. Frequency of no response was 2.

Table 4: Error Types, Rates, and Frequencies for Items 33 and 34

Error types	Examples	Rates %	Frequencies
Misunderstanding of the place value (Joung & Kim, 2022)	4.3 × 0.2 = 8.6	83.3	24
Misunderstanding of mathematical equivalence (Joung & Kim, 2022)	1.2 × 3 = 3.2, 1.2 × 3 = 1.6	73.3	30
Inappropriate application of addition	4.3 × 0.2 = 4.5	3.3	1
Other	4.3 × 0.2 = 4.3	6.7	2

Note. Rates and Frequencies indicate the proportion of students who made the error and the number of occurrences of that error respectively. Frequency of no response was 1.

For Items 37 to 46, students continued to use counting strategies and additive calculation strategy; however, errors caused by counting mistakes were relatively infrequent. In contrast, almost all observed errors were consistent with those reported in previous studies (Tables 5, and 6). Based on the errors observed in Items 39 to 41, it became evident that more than half of the students may not understand or be able to correctly interpret the fundamental concepts of fractions and fraction addition and subtraction (Mutiara, 2025). In Items 42 and 46, errors involving errors resulting from failing to convert the divisor as well as errors involving the inappropriate application of addition rules to multiplication and division were also observed, providing further evidence that students lacked a proper understanding of the fundamental concepts of fractions (Lin et al, 2025). These findings are consistent with previous research indicating that Grade 9 students in Soweto, South Africa, hold conceptual misunderstandings regarding fractions (Moyo & Machaba, 2022). In addition, errors reflecting misunderstandings of mathematical equivalence, such as calculating 5 × 0.3 as 0.15 (Joung & Kim, 2022), as well

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as errors such as calculating 5 ÷ 9 as 0.2, were also observed, further revealing students' incomplete understanding of decimal multiplication and division. Several error types not reported in previous studies were also identified. For example, some students incorrectly calculated 5 × 0.3 as 5.03.

Table 5: Error Types, Rates, and Frequencies for Items 37 and 38

Error types	Examples	Rates %	Frequencies
Mistakes due to carelessness or lack of thoroughness (Mutiara, 2025)	$\frac{5}{9}+\frac{2}{9}=\frac{3}{9}$	6.7	2
Operation errors (Mutiara, 2025) or Counting mistakes	$\frac{8}{9}-\frac{2}{9}=\frac{5}{9}$	6.7	2

Note. Frequencies indicates the number of occurrences of that error.

Table 6: Error Types, Rates, and Frequencies for Items 39 to 41

Error types	Examples	Rates %	Frequencies
Conceptual errors (Mutiara, 2025)	$\frac{1}{2}-\frac{2}{5}=\frac{2}{10}$, $\frac{3}{4}+\frac{1}{6}=\frac{4}{10}$, $\frac{1}{2}-\frac{2}{5}=-\frac{1}{10}$, $\frac{1}{4}+\frac{2}{2}=\frac{3}{2}$, $\frac{1}{2}-\frac{2}{5}=\frac{5}{10}-\frac{4}{10}=1$	56.7	40
Principle errors (Mutiara, 2025)	$\frac{3}{4}+\frac{1}{6}=\frac{3\times6}{4\times6}+\frac{1\times6}{6\times6}=\frac{18}{24}$	6.7	4
Mistakes due to carelessness or lack of thoroughness (Mutiara, 2025)	$\frac{3}{4}+\frac{1}{6}=\frac{18}{24}-\frac{4}{24}=\frac{14}{24}$	10.0	3
Operation errors (Mutiara, 2025) or Counting mistakes	$\frac{3}{4}+\frac{1}{6}=\frac{18}{24}+\frac{4}{24}=\frac{20}{24}$	10.0	3

Note: Rates and Frequencies indicate the proportion of students who made the error and the number of occurrences of that error respectively. Frequency of no response was 10.

Responses to the Research Questions and General Discussion

In this section, based on the descriptive statistics presented in these sections and the error analysis presented, responses to Research Question I and Research Question II are provided, followed by a comprehensive discussion based on these findings.

First, Research Question I is addressed. The correct response rates for the tests on addition, subtraction, and basic multiplication were relatively high, at 79.3, 68.0, and 70.0 points, respectively, whereas the correct response rates for multiplication and division were lower, at 48.0 and 60.0 points, respectively. Students' complete reliance on counting strategies was observed, and no selection of more efficient strategies, such as decomposition strategy or retrieval strategy, was observed. The primary cause of errors was counting mistakes, and even in situations where students employed additive calculation strategy, column method, or commutative property, they were essentially relying on counting strategies. As a result, errors involving overcounting or undercounting, as well as ±1 or ±2 deviations, frequently occurred. Furthermore, errors such as calculating 20 × 30 as 60 or 40 × 800 as 3,200 suggest that students' conceptual understanding of place value may be fragile. Taken together, three major issues were identified: reliance on counting strategies, failure to acquire high-efficiency strategies, and insufficient conceptual understanding of place value. These issues were found to be so severe that they substantially hinder both the accuracy and speed of multi-digit calculations, even among academically high-achieving students within the school. Among these issues, reliance on counting strategies is considered the fundamental problem, as it likely induces both the failure to acquire high-efficiency strategies and the insufficient conceptual understanding of place value (Geary et al., 2004; Kerkman & Siegler, 1993; Laski et al., 2013; Lemaire & Callies, 2009; Siegler, 1988; Uchida, 2009).

Next, Research Question II is addressed. In the addition and subtraction of decimals and in the addition and subtraction of fractions with common denominators, correct response rates were relatively high, exceeding 80%, and no critical errors were observed. In contrast, correct response rates were markedly low for decimal multiplication and division, fraction addition and subtraction with unlike denominators, and fraction multiplication and division. In particular, the correct response rates for Item 34, multiplying decimals, Item 43, multiplying a fraction by a decimal, Item 44, dividing a fraction by an integer, and Item 46, dividing a fraction by a decimal, were extremely low, at 3%, 13%, 13%, and 0%, respectively. In the decimals, errors were mainly attributable to counting mistakes, misapplication of the right justification, misunderstanding of the place value, and misunderstandings of mathematical equivalence. In the fractions, errors were mainly attributable to conceptual errors, principle errors, and operation errors. Based on these error analyses, it was revealed that more than 80% of the students may have difficulties with the conceptual understanding of decimal place value, and more than 70% of the students may lack sufficient knowledge of mathematical equivalence. In addition, it was suggested that more than half of the students did not understand the fundamental concepts of fractions. When these findings are synthesized together with the results for Research Question I, it is suggested that students' reliance on counting strategies inhibits their recognition of numbers as composite units and their conceptual understanding of place value, thereby preventing them from attaining an understanding of the fundamental concepts of decimals and fractions. Notably, this issue was found to be severe even among high-achieving students.

Based on these results, it can be concluded that helping students disengage from reliance on counting strategies is the most critical issue. The ability to perceive numbers as composite units and to understand place value concepts constitutes a foundational competence for the development of higher-order computational skills (Uchida, 2009), mathematical concept development, and ultimately abstraction and generalization abilities (Mulligan et al., 2015). In the present study, these issues were found to be particularly pronounced in the domains of decimals and fractions. This tendency is consistent with observations reported by the Japan International Cooperation Agency (2024). The fact that students with difficulties in conceptual understanding were nevertheless able to answer some decimal and fraction problems correctly suggests that they may have blindly applied procedural approaches without conceptual understanding (Moyo & Machaba, 2022). The misapplication of addition rules to multiplication and division in fraction provides further support for this interpretation.

Why, then, do South African students rely so heavily on counting strategies? To explore this question, the author analyzed the South African curriculum (Department of Basic Education, 2011a; 2011b; 2011c) as well as textbooks sponsored by Sasol, a chemical and energy company that provides financial and capacity-building support to educational institutions (The Ukuqonda Institute, 2016a; 2016b; 2016c; 2017a; 2017b; 2017c). Students are expected to learn a calculation method known as the "Breaking down and building up numbers method," which focuses on units of ten, beginning in Grade 1. However, classroom observations indicated that instruction emphasizing units of ten was insufficient, and that even after decomposing numbers, instruction ultimately encouraged students to solve problems using counting strategies. This instructional practice may be one factor contributing to students' limited improvement in basic computational skills. It has been pointed out that unless instruction fosters recognition of numerical groupings, students continue to rely on counting strategies even as they progress through grade levels (Nakawa, 2016). In addition to column method, the textbooks present a variety of calculation methods, such as the "Doubling and halving numbers method," which uses doubling and halving, "Rounding off and compensating numbers method," which uses estimation, and even calculator-based methods. In the curriculum for Grades 4 to 9 (Department of Basic Education, 2011b; 2011c), these diverse methods are intended to promote the selection of optimal strategies according to the problem and the verification of answers using alternative methods. However, interviews with teachers at neighboring primary schools indicated that the large number of calculation methods may instead cause confusion among students, potentially contributing to poor basic computational performance.

Previous research has shown that emphasizing conceptual understanding of the base-ten structure from the early stages of mathematics instruction is particularly beneficial for the development of children's computational abilities (Herzog et al., 2017). It has also been suggested that supporting students' recognition of mathematical patterns and structures enhances overall mathematical competence (Mulligan & Mitchelmore, 2013). As specific instructional approaches, it has been suggested that enabling learners to experientially understand the structure of the base-ten system through the use of semi-concrete materials such as bottle caps (Nakawa et al., 2020) is effective. In addition, lesson designs that promote attention to and use of mathematical structures through modeling activities based on gear tooth ratios and multiplicative relationships have also been shown to be effective (Deis & Julius, 2017). Furthermore, the use of Structuring Number Starters, developed through research conducted in South Africa by Venkat et al. (2021), can also be considered. This is a practice package intended to support learners' transition from reliance on counting strategies to calculations based on relationships and structures among quantities.

CONCLUSION AND RECOMMENDATIONS

Results from TIMSS and national learning assessment tests and surveys conducted by the Japan International Cooperation Agency (2024) have emphasized low levels of learning achievement among students from primary education through lower secondary education, with attention to insufficient basic computational skills and conceptual understanding at the primary education stage. The present study was initiated to investigate the actual situation of upper secondary school students with respect to these issues and to clarify how their basic computational skills affect their learning of mathematics. Accordingly, the survey items were first subdivided, and a paper-based test was developed and administered. To reduce errors caused by carelessness or incomplete execution and to accurately capture students' errors, a fixed response time was set to ensure that students' abilities could be clearly measured. By evaluating, in an integrated manner, tasks related to solution methods for the four basic operations involving integers, decimals, and fractions, the present study comprehensively assessed students' computational abilities and reconfirmed the importance of instruction in computational strategies and conceptual understanding at the primary education stage. In addition, this study was conducted with students from the high-achieving group. By analyzing their learning status in detail, the results suggest that the challenges faced by students in other achievement groups may be even more severe. Furthermore, by conducting the investigation in a different province of South Africa that had not been sufficiently examined in previous research, this study was able to provide insights that contribute to a broader understanding of students' academic achievement across South Africa.

However, the present study did not extend to visualizing students' cognitive processes in detail. In addition, unanswered items and certain errors that were inconsistent with previous research were not examined sufficiently. These limitations suggest several directions for future research. First, more fine-grained analyses employing think-aloud protocols and cognitive measures are needed. Such approaches would provide deeper insight into the mechanisms underlying students' strategy selection and errors. Furthermore, studies involving larger and more diverse samples are necessary to enhance the generalizability of the findings.

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