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An Analysis of the Ornstein-Uhlenbeck Process Application to Generate a Pricing Formulae for Tea Derivatives in Nduti Tea Factory.

[1*]Patrick. MUMU
Undergraduate Student BSC Actuarial Science
Nairobi University
*Corresponding Author's Email: pat7mumu@gmail.com

Abstract

Purpose: The purpose of this study was to analyse a suitable and acceptable model to generate a pricing formulae for tea derivatives.

Methodology: The study used descriptive survey research design. This study used secondary data which was collected from Nduti tea factory website. The target population of the study were 318 auction days' observations on the stock exchange spread over from 18/12/2007 to 2/12/2014. Purposive sampling was used to select 6 working days excluding Sundays and holidays starting from 18/12/2014 to 2/12/2014.Data from the websites was analysed using the Ornstein Uhlenbeck process, to derive descriptive results.

Results: The findings implied that risk mitigation is an important element that is considered in price derivatives. The study findings indicated that Ornstein-Uhlenbeck Process application to generate a pricing formulae mitigates the risk presented by the fluctuating prices. Further pricing derivatives facilitates the trading of various commodities as a financial instrument.

Unique contribution to theory, practice and policy: The study provides the importance which will be emphasized on insuring farmers from uncertainty by ensuring they get value for the input and costs of production. On the other hand, consumers will be protected from the volatile food commodity prices. An incentive for the farmer is established and hence increased and more efficient productivity is witnessed. This leads to designing simple commodity derivatives with different times to expiry for tea in Kenya based on estimated future market prices. The results of this study will be of particular significance to farmers, cooperatives and general investors. The derivatives model proposed in this study can be adopted and used in the agricultural industry especially with regard to tea.

Keywords: *price derivatives, Ornstein Uhlenbeck process, commodity derivative, forwards, futures, options or swaps.*

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1.1 INTRODUCTION

A commodity derivative contract exists where an individual acquires the right to buy or sell a commodity for a certain price on a specified date, at a margin price (price of the contract) from a contract seller who accepts this margin. These derivatives may take the form of forwards, futures, options or swaps.

Future contracts serve many purposes. Their first role has been facilitating the trading of various commodities as a financial instrument. But they have from the start been providing a hedging vehicle against price risk (Helyette geman, 2009). He further goes on to explain that a farmer selling his crops in January through a future contract maturing at time T of the harvest, for a price $F^T(0)$ defined on 1st January has secured at the beginning of the year this amount of revenue. Hence, he may allocate the proceeds to be received to the acquisition of new machinery or storage facilities and, more generally, design his investment plans for the year.

The derivatives market was first introduced in a bid to mitigate the risk of farmers selling their produce below cost price. Commodity markets may either be in form of exchange traded or over the counter (OTC) trading, or both. Off-exchange trading, better known as over the counter (OTC) trading consists of contracts performed directly by two parties without external supervision. The main participants in the futures market includes the hedgers, speculators, and arbitragers. Hedging through the use of derivatives is crucial in many sectors of the economy due to the changing and volatile nature of the world market. Arbitragers are investors who attempt to profit from price anomalies by trading in simultaneous transactions that offset each other and in the process acquiring risk-free profits. Speculators try to anticipate price movements with the hope of making a profit. Hedgers are viewed as risk averse while speculators are more prone to taking risks.

Trading commodities can be grouped into four major categories namely energy, metals, agricultural, livestock and meat. In the course of our project, we will deal with tea derivatives which are part of agricultural commodity derivatives

1.2 Problem Statement

Pests and diseases, climatic hazards (unreliable rainfall), delayed payments among other factors have led to price fluctuations in the tea industry. As a result, farmers are forced into settling for meagre proceeds and thus this weakens their morale in cultivation of the cash crops. Such farmers may tend to abandon tea in favour of other appealing ventures or cease farming altogether.

According to The Business Daily, the price of tea at the Mombasa auction hit a three-month low towards the end of the year 2014. As a result, more than half a million farmers who sell their tea through Kenya Tea Development Agency (KTDA) were forced into earning lower bonuses. The tea board of Kenya, responsible for regulating the market, blamed unsold tea from previous auctions as having caused the price dip.

A system should be put in place so as to mitigate the risk presented by the fluctuating prices. The problems that this study aims to solve or at least control (to a considerable extent) include among others: Uncertainty- Control fluctuating prices by establishing a specified price over a period of time, based on future expected spot prices of tea, Deteriorating Economic Conditions- Discourage manufacturers from importing tea while at the same time providing investment opportunities for local investors ,Demand Supply Deficit- Give an incentive to tea farmers to produce and store more to meet demand, by assuring a fixed reasonable price and constant demand for produce and finally Transparency- Determine prices purely on the basis of demand and supply.

1.3 Study Objective

- To fit a suitable and acceptable model to generate a pricing formulae for tea derivatives.

2.0 LITERATURE REVIEW

Carlos Blanco (2001) using Geometric Brownian motion model concluded that the speed at which prices revert to their long run levels may depend on several factors such as the nature, magnitude and direction of the price shock. If we calibrate the mean reversion rate for each month of the year using data exclusively from that month, we would find that for most markets the mean reversion rates differ considerably. The limitation on this model is that it has more complex processes that incorporate more information about the possible price changes, but at the cost of having to estimate many more parameters and increasing the probability of model errors. The only unknown parameter in the Geometric Brownian motion is the volatility of the underlying. The ideal volatility to use for modelling purposes would be the 'future volatility', but by definition, it is not possible to know the 'future volatility' until we know what has happened in the market, and by that time has become 'historical volatility'. Any trader knows that volatilities change through time, and the assumption of constant volatilities may not be very realistic. More complex processes incorporate time varying volatility, and some Risk Management models assume that volatilities fluctuate just as asset prices do Olaf Korn (2005) generalizes the model proposed by Schwartz and Smith (2000). The model allows for two mean-reverting stochastic factors and therefore implies that spot and futures prices can be stationary

Futures prices according to the Schwartz (1997) and Schwartz and Smith (2000) models reveals that when taking the appropriate limits of the relevant parameters, the long-term price level follows a mean-reverting process and the sensitivity of the futures price for changes in decreases with the time to maturity of the contract. However, if the process of the long-term level is not meanreverting, the sensitivity is the same for all maturities. Thus the above models suggest that the stationary or non-stationary element of the spot price process is particularly important for the pricing and hedging of contracts with a long time to maturity.

Ross (1981) concluded that the futures price is not a function of spot volatility, jumps or their associated parameters. It should be emphasized, however, that this result does not imply that stochastic volatility and jumps are unimportant. Future price is the expectation of the future spot price under the risk neutral measure. While stochastic volatility and jumps affect the higher-order moments of the terminal spot price distribution, they do not alter the first moment. In contrast, it will be shown that stochastic volatility and jumps play critical roles in pricing options. This is because option prices are sensitive to higher-order moments of the distribution. Coughenour, Seguin, and Smoller (1995) showed seasonal patterns. They argued that Standard no-arbitrage completely determine the drift of the price processes under risk-neutral probabilities leaving no room for explicit modelling of mean reversion or seasonal effects via the drift of the spot commodity price. However, the spot convenience yield process enters the drift of the spot commodity price under risk- neutral probabilities. Hence, mean reversion and seasonal behaviour is possible by manipulating volatilities and the correlation structure of the joint process of spot commodity prices and convenience yields. For example, a positive correlation between the spot commodity price and the spot convenience yield will have a mean reversion effect on the spot commodity price even under risk-neutral probabilities. It is worth to mention that the futures and forward price coincide since in their model the interest rate is assumed to be constant.

3.0 RESEARCH METHODOLOGY

The Simulation and Estimation of the Ornstein-Uhlenbeck Process and the Application of the Process to Commodities Markets and Modelling is a mean reversion process used extensively in finance to model interest rates and also by those who model commodities. The Ornstein and Uhlenbeck (1930) ('O-U') process, also referred to as the Vasicek (1997) process is the most popular model for such work.

Mean reversion processes are generally attractive to model assets because they incorporate the economic argument that when prices are excessively high, demand reduces and supply increases which has a counterbalancing effect. When prices are low, the reverse occurs and prices plummet to some sort of long-term mean.

The Ornstein Uhlenbeck process, S, is modelled as follows:

$$ds = \lambda(\mu - S)dt + \sigma dW_t$$

Where;

σ, is the measure of volatility λ, is the

measure of speed of mean reversion

W_t, is the Brownian Motion, hence $W_t \sim N(0,t)$ μ, is the long-

term mean to which the process tends to revert.

This process has well known closed form solutions and has 3 parameters to approximate which are the estimators for α, μ and σ. Its downside is that nothing prevents the process from going negative. In case this is unwanted, two things can be done:

Revise the process away from a pure O-U process and harmonize the volatility parameter as S approaches zero, for instance, the Cox, Ingersoll, Ross (1985) model alterations in interest rates 'r' as:

$$dr = \kappa(\theta - r)dt + \sigma\sqrt{r}dW_t$$

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Model the log of the spot price, so that a log-spot of below zero corresponds to a spot price that is above zero.

Estimating the Parameters of an Observed Ornstein-Uhlenbeck Process

The popular techniques for estimating the parameters are Maximum Likelihood and Least Square Regressions.

Ordinary Least Squares Estimation – One Factor Vasicek Model on log Spot Price

With this method as per Roelf Sypkens (2010), based on the log spot prices the model is expressed in the following simple linear equation.

$$Y = mX + c + \varepsilon$$

Minimizing the variance of, ε, the error term, we obtain the estimates: û, The

basic one-factor Vasicek model is defined as

$$dP_t = \alpha(\mu - \ln P_t)P_t dt + \sigma P_t dW_t$$

Where:

α, is the mean reversion rate W_t, is

a standard Wiener process σ, is the

volatility of the spot prices

μ , is the related long-term level of the natural log of the spot prices The

above equations yield:

$$\alpha = -\frac{\ln(m+1)}{\Delta t}$$

And

$$\mu = \frac{c}{\left(1 - e^{-\alpha\Delta t}\right)} + \frac{\sigma^2}{2\alpha}$$

The following are the estimates calculated using the natural logarithm of the tea spot prices and thus the parameters α, μ and σ^2 need to be estimated so that their optimal values can be obtained.

Minimizing the Variance of the Error Term

$$\hat{\sigma}^2 = \frac{2\hat{\alpha}}{\left(1-e^{2\hat{\alpha}\Delta t}\right)}\left(\sigma_Y^2 - \frac{\sigma_{XY}^2}{\sigma_X^2}\right)$$

Estimation using the Maximum Likelihood Method (M.L.E) From the equation:

$$x_{t_{i+1}} - x_{t_i} = x_{t_i}\left(e^{-\alpha\Delta t}-1\right)+\left(\mu-\frac{\sigma^2}{2\alpha}\right)\left(1-e^{-\alpha\Delta t}\right)+\sigma e^{-\alpha t_{i+1}}\int_{t_i}^{t_{i+1}} e^{\alpha s}dW_s$$

We obtain the conditional expectation and variance, respectively are as shown below:

$$v_{t_{i+1}} = E\left(x_{t_{i+1}} \mid x_{t_i}\right)$$

$$v_{t_i} = E\left(x_{t_{i+1}} \mid x_{t_i}\right)$$

$$= E\left(x_{t_i}e^{-\alpha t_i}+\left(\mu-\frac{\sigma^2}{2\alpha}\right)\left(1-e^{-\alpha t_i}\right)+\sigma e^{-\alpha t_i}\int_0^{t_i} e^{\alpha s}dW_s\right)$$

$$= E\left[x_{t_i}e^{-\alpha t_i}\right]+E\left[\left(\mu-\frac{\sigma^2}{2\alpha}\right)\left(1-e^{-\alpha t_i}\right)\right]+\sigma e^{-\alpha t_i}E\left[\int_0^{t_i} e^{\alpha s}dW_s\right]$$

$$= x_{t_0}e^{-\alpha t_i}+\left(\mu-\frac{\sigma^2}{2\alpha}\right)\left(1-e^{-\alpha t_i}\right)$$

Because:

$$E\left[\int_0^{t_i} e^{\alpha s}dW_s\right]=0$$

And finally,

$$w_{t_i}^2 = \sigma^2 e^{-2\alpha t_i}E\left[\int_0^{t_i} e^{\alpha s}dW_s\right]^2$$

$$= \sigma^2 e^{-2\alpha t_i}\int_0^{t_i} e^{2\alpha s}ds$$

$$= \frac{\sigma^2 e^{-2\alpha t_i}}{2\alpha}\left[e^{2\alpha t_i}-1\right]$$

$$= \frac{\sigma^2}{2\alpha}\left[1-e^{-2\alpha t_i}\right].$$

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We, therefore, take the log spot prices to be normally distributed such that $x_{t_i} \sim N(v_{t_i}, w_{t_i}^2)$ for i = 1,2,3…n

Defining a likelihood function,

$$L(\alpha, \mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi w_i^2}} \exp\left(-\frac{1}{2w_{t_i}^2}(x_{t_i} - v_{t_i})^2 \right)$$

Its natural logarithm is:

$$\ln L(\alpha, \mu, \sigma^2) = -\frac{n}{2}\ln 2\pi - \sum_{i=1}^{n}\left(\frac{\ln w_{t_i}^2}{2} + \frac{1}{2w_{t_i}^2}(x_{t_i} - v_{t_i})^2 \right)$$

Differentiating the natural logarithm of the likelihood function with respect to α, μ and $\hat{\alpha}$ then setting the derivative to zero, yields the following MLEs.

$$\hat{\mu} = \frac{S_y - e^{-\hat{\alpha}\Delta t} S_x}{n(1 - e^{-\hat{\alpha}\Delta t})}$$

$$\hat{\alpha} = -\frac{1}{\Delta t}\ln\left(\frac{[S_{xy} - \hat{\mu}S_x - \hat{\mu}S_y + n\hat{\mu}^2]}{S_{xx} - 2\hat{\mu}S_x + n\hat{\mu}^2} \right)$$

$$\hat{\sigma}^2 = \frac{2\hat{\alpha}}{n(1 - e^{-2\hat{\alpha}\Delta t})}\left[S_{yy} - 2e^{-\hat{\alpha}\Delta t} S_{xy} + e^{-2\hat{\alpha}\Delta t} S_{xx} - 2\hat{\mu}_t(1 - e^{-\hat{\alpha}\Delta t})(S_y - e^{-\hat{\alpha}\Delta t} S_x) + n\hat{\mu}_t^2 (1 - e^{-\hat{\alpha}\Delta t})^2 \right]$$

The Normal No- Arbitrage Pricing Equation

As we price these commodity derivatives, we make the assumption that the market is arbitrage free, meaning, if there aren't initial wealth and risk is not taken, the ultimate payoff should be null. Under this assumption, it can be laid out that the forward price, with time, T, to maturity of a commodity is actually given by:

$$f^T(t) = S(t)e^{r(T-t)}$$

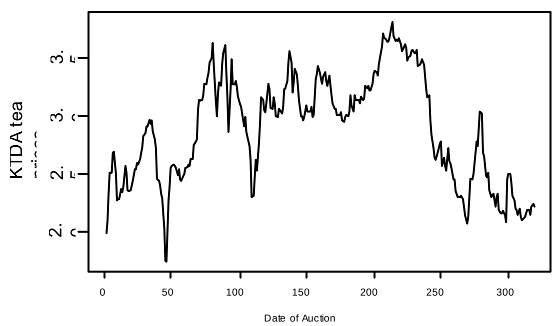
Where, r, is the prevailing continuously compounded interest rate at time t. It is expected to remain constant over the period

4.0 RESULTS AND DISCUSSIONS

A graphical trend of the KTDA tea prices indicates the possibility that the process could be a random walk process. A random walk process is usually an AR(1) process. However, further tests are required in order to confirm whether the data set follows an AR(1), AR(2) or an ARMA(1,1) process.

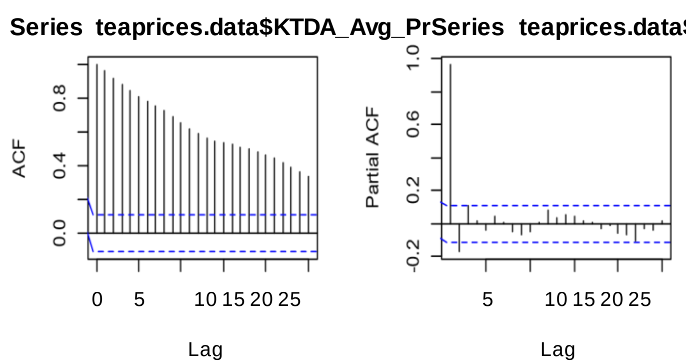
Steps in model building;

Identify the kind of model (p,q) by running ACF and PACF. Use simulated models to compare the ACF and PACF and finally estimate the coefficients. Carry out diagnostics to prove that the residuals follow a white noise process. The various diagnostics for residuals include QQ plot, ljung box pierce statistics, rank test, sign test, AIC and BIC

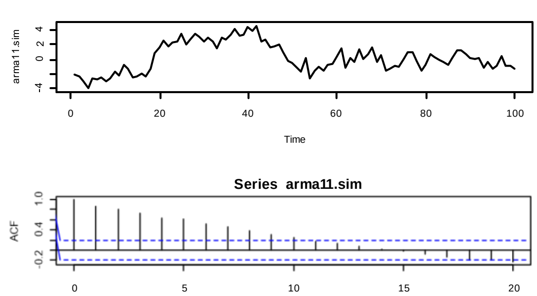


Autocorrelation Function and Partial Autocorrelation Functions

The ACF of tea prices indicates that the process is non decaying, the PACF indicates 2 bars, a positive one and a negative



Simulations to enable model identification AR(1) process: ACF and PACF



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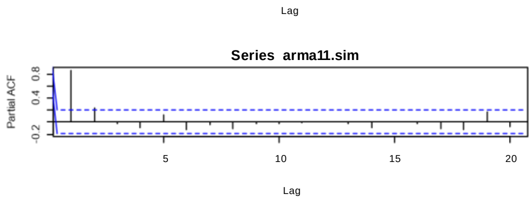

The no decaying ACF is similar to the tea prices data set, the PACF has two significant spikes could be an indicator that this is an ARMA (1, 1) process.

Model Estimation (ARMA(1,1)

The ARMA(1,1) model can be estimated in MLE

$X_t= 0.9641X_{t-1}+0.2955\, w_{t-1}+2.7662+ w_t$ sigma^2

estimated as 0.01051 Summary of Results

Alpha(α)	Theta(θ)	Intercept/Mu(μ)	Sigma2(σ) MLE
0.9641	0.2955	2.7662	0.01051

Residual diagnostics

The trend of the standardized residuals indicates that the residuals follow a white noise process. The ACF for residuals show that the covariance of residuals is zero and hence they follow a white noise process. The normal QQ plot indicates that the deviations from the line of best fit are small, hence no outliers and this is an indicator that the residuals follow a white noise process. The p values for L jung statistic indicate that the null hypothesis (white noise process) is not rejected and hence the residuals follow a white noise process.

5.0 DISCUSSION CONCLUSIONS AND RECOMMENDATIONS

The study objective was to analyse a suitable and acceptable model to generate a pricing formulae for tea derivatives. This is because The Normal No- Arbitrage Pricing Equation assumes that we make the assumption that the market is arbitrage free, meaning, if there aren't initial wealth and risk is not taken, the ultimate payoff should be null. Results show that sigma^**2** estimated as 0.01051 show variations as compared to the non-arbitrage model which assumes there are no fluctuations.

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