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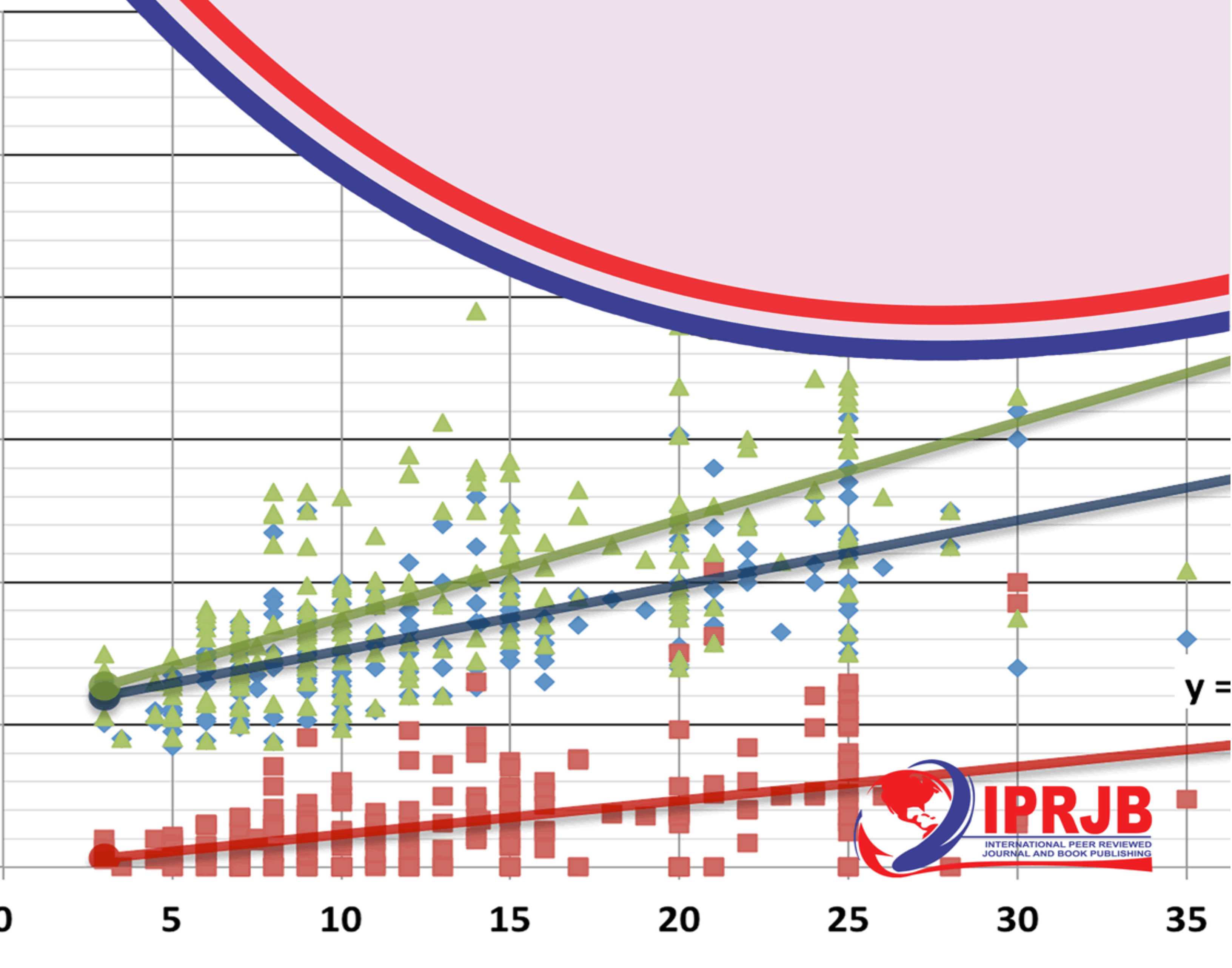

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LOCATION OF ADDITIONAL LIBRARY FACILITY IN BEREKUM MUNICIPALITY USING BERMAN AND DREZNER ALGORITHM

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Purpose: To model location of an additional library facility in the Berekum Municipality as a conditional *p*-center problem which will serve as a reference centre for Schools and Colleges within the municipality

Methodology: The data for this study was the road distance between the suburbs of Berekum Municipality. The suburbs of the municipality were coded and Floyd's algorithm was used to find the distance matrix, d (i, j) for all pairs shortest path. Subsequently, the Researchers used Berman and Drezner's algorithm on 18-nodes network which had two existing library facilities in Berekum and Jininjini to locate additional Library facility for Library users in Berekum Municipality.
Matlab program software was used for the coding of the Floyd-Warshall algorithm. The codes for Floyd-Warshall algorithm was developed and ran on DellAMD Athlon (tm) II P360 Dual-Core Processor 2.30GHz of RAM 3.00GB, 64-bit Operating System with Windows Ultimate Laptop Computer

Results: The analysis of the study revealed that, an additional library facility using Berman and Drezner (2008) should be located at Akrofro with an objective function value of 8. The results obtained from the study is useful to locate a public library facility that will benefit all the people in Berekum municipality. Specifically, the results revealed the new public library facility to be built in Berekum Municipality should be sited at Akrofro.

Unique contribution to theory, practice and policy: The implication of the results is that the minimum distance travelled by the farthest library user to the new library facility at Akrofro is 8 kilometres. Stakeholders should adopt the use of Berman and Drezner's algorithms in establishing facilities such as markets, hospitals, recreational centres and so on.

Keywords: *Algorithm, p-centre, location problem, library facility, Berman and Drezner.*

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INTRODUCTION

A library is a considerable collection of books kept for use and not as merchandise. It is organised to be used and maintained by a public body, an establishment, or a personal individual. In a more traditional sense, UNESCO (1994) defined the public library as the local access to knowledge, provides a basic condition for lifelong learning, independent decision making and cultural development of the individual and social groups. A country's public libraries happen to serve the widest population possible. The role of the library in education has long been acknowledged and is considered to be crucial because a library often provides a place of silence for studying. (Wikipedia, 2012). Public and private libraries are intended for use by people who choose not to, or cannot afford to buy an extensive collection of reading materials themselves. Human access to relevant and current information is crucial for the economic and social well-being of humans. Libraries for ages are known for providing information and promoting knowledge. A library is a corporation whose sole aim is to gather information, process it, preserve it and eventually broadcast it in various formats most convenient to its users (Ogbah, 2014). Libraries exist for people, and not the reverse. Libraries mainly exist for the purpose of maintaining records of human knowledge and civilization. Without libraries, human philosophies would not have been successfully transferred across generations (Abbas, 2012). In order to successfully withstand their cultural roles, libraries have been classified into different types to discharge their services better and to please more patrons (Obinyan; Obinyan & Aidenojie, 2011). According to Parvathamma and Reddy (2009), public libraries are known to develop literacy through numerous information and academic services they render over time.

Statement of the problem

Due to the nonexistence of libraries in most communities in Berekum Municipality, the performance of students is low. Where there are libraries, the locations are poorly sited which most library users don't get access to the library facility.

The researchers, therefore, intend to locate a further library facility to serve the people of Berekum Municipality who had to travel an extended distance to any of the sole two existing libraries (one at Berekum and the other at Jinijini) and also people who do not have access to the library at all.

Objectives of the Study

To the model location of an additional library facility in the Berekum Municipal as a conditional *p-centre* problem

To solve the conditional *p-centre* problem using Berman and Drezner's algorithm

LITERATURE REVIEW

To locate a specific type of facility, one usually looks for the best way to serve a set of communities whose location and demands are known. This implies one needs to consider the following:

1. The number and location of the facilities to serve the demand
2. Size and capacity of each facility
3. The allocation of the demand points to open facilities

4. Optimizing some objective location function.

Most location models deal with desirable facilities, such as warehouse, service and transportation centres, emergency services, etc, which interact with the customers and where distance travel is involved. As a consequence, typical criteria for such decision include minimizing some function of the distance between facilities and/ or clients (i. e., average travel time, average response time, the cost function of travel or response time, maximum travel time or cost, etc.).

The traditional optimality criterion of closeness (to locate the facility as close as possible to the customers) is replaced by the opposite criterion of how far away from the customers can the facility be placed to ensure accessibility to the demand point. This generates the NIMBY syndrome (NOT-IN-MY-BACK-YARD) (Capitivo & Climaco, 2008). Buettcher (2004), described the p-centre problem, as the Min-Max Multicenter problem or the Facility Location Problem, to be a famous problem from operations research. Buettcher classified the optimization problem into three different types, depending on which of the restrictions applied as follows:

i. The general optimization problem in which the choice of the distance function d is not restricted in any way.

ii. The metric problem in which d satisfies the triangle inequality.

iii. The metric and symmetric problem in which d(x; y) = d(y; x), and d satisfies the triangle inequality.

It was realized that the metric, asymmetric p-centre problem had remained unstudied even ten years after its symmetric counterpart had been finally solved (by presenting an algorithm with optimal approximation factor) in 1986. In 1998, the O (log*(n)) approximation algorithm found by Panigrahy and Vishwanathan (1998) was published. Thus, it was clear that in contrast to the general p-centre problem without any restrictions to the distance function-this problem could be approximated. And the approximation was a very good one, although the algorithm could not guarantee a constant-factor approximation.

Conditional Location Problem

The conditional location problem is to locate p new facilities to serve a set of demand points given that q facilities are already located. When q is equal to zero (q = 0), the problem is unconditional. In conditional p-centre problems, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand (Berman, 2008). The p-centre problem seeks the location of p facilities. Each demand point receives its service from the closest facility. The objective is to minimize the maximal distance for all demand points. The p-centre problem consists of choosing p facilities among a set of M possible locations and assigning N clients to them to minimize the maximum distance between a client and the facility to which it is allocated.

Elloumi et al. (2004), presented a new integer linear programming formulation for this min-max problem with a polynomial number of variables and constraints, and show that its LP relaxation provides a lower bound tighter than the classical one. Moreover, they showed that an even better lower bound LB*, obtained by keeping the integrality restrictions on a subset of the variables, can be computed in polynomial time by solving at most O (log2 (NM)) linear programs, each having N rows and M columns. They also showed that when the distances satisfy triangle inequalities, LB* is at least one-third of the optimal value. Finally, they used the LB* in an exact solution method and report extensive computational results on test problems from the literature. For instances where the triangle inequalities are satisfied, their method outperforms the running time of other recent exact methods by an order of magnitude. Also, it is the first one to solve large instances of size up to N = M = 1, 817.

Krumke (1995), considered the generalization of the p-Center Problem, which is called the α-Neighbor p-Center Problem ( ). Given a complete edge-weighted network, the goal is to minimize the maximum distance of a client to its α nearest neighbour in the set of p centres. He shows that in general finding an O (2 poly(¦V¦))-approximation for is NP-hard(Garey and Johnson, 1979), where ¦V¦ denotes the number of nodes in the network. If the distances are required to satisfy the triangle inequality, there can be no polynomial-time approximation algorithm with a ( ) performance guarantee for any fixed and any fixed α, unless P = NP. For this case, He presented a simple yet efficient algorithm that provides a 4-approximation for α 2. Considering the p-centre Problem with Connectivity Constraint, let G (V, E, W) be a graph with n-vertex-set V and m-edge-set E in which each edge e is associated with a positive distance W (e).

Chung-Kung and Chein-Tsai. (2006), proposed an additional practical constraint which restricted the p vertices, to be connected. The resulting problem is called the connected p-centre problem (the CpC problem). The first show that the CpC problem is NP-Hard on bipartite graphs and split graphs. Then, an O (n)-time algorithm for the problem on trees is proposed. Finally, the algorithm was extended to trees with forbidden vertices. That is some vertices in V cannot be selected as centre vertices, and the time-complexity is also O (n). Meanwhile, it was identified that other variants of the traditional p-Center problem are also a very important issue. For example, just restricting that the p-centre must be “total”, thus, the subgraph induced by the p-centre has no isolated vertices, is another typical practical variant.

Chen and Chen (2009), presented a new relaxation algorithm for solving the conditional continuous and discrete p-centre problems. In the continuous p-centre problem, the location of the service facilities can be anywhere in the two-dimensional Euclidean space. In the discrete variant, there is a finite set of potential service points to choose from. An analogous representation of the discrete p-centre problem is the p-centre problem on networks. In the p-centre problem on networks, both the demand points and the potential service points are located on a weighted undirected graph, and the distance between any two points is the cost of the shortest path between them. They assumed that there are a finite number of values for the optimal solution of an unconditional p-centre problem. They use the assumption to implement the subroutine Get- Next Bound (Lower-Bound) which returns the smallest value, among the possible values for the optimal solution, which is greater than Lower-Bound. Also, the subroutine Find Feasible Solution (Sub, r), which answers the question: "Is there a solution to the sub-problem with a value less than r?" (And if so, finds such a solution).

Hassin et al. (2003), introduced a local search strategy that suits combinatorial optimization problems with a min-max (or max-min) objective. According to this approach, solutions are compared lexicographically rather than by their worst coordinate. They apply this approach to the p-centre problem. Based on a computational study, the lexicographic local search proved to be superior to the ordinary local search. This superiority was demonstrated by a worst-case analysis.

Minieka (2006), stated that previous treatments of location problems on a graph have been confined to the optimum location of a single facility or the simultaneous optimum location of multiple facilities. The author addresses the problem of optimally locating a facility on a graph when one or more other facilities have already been located in the graph. The author shows that previous solution techniques can be reused if the distances in the graph are judiciously redefined.

Tamir et al. (2005), deal with the location of extensive facilities on trees, both discrete and continuous, under the condition that existing facilities are already located. They require that the selected new server is a subtree, although we also specialize to the case of paths. They study the problem with the two most widely used criteria in Location Analysis: centre and median. Their main results under the centre criterion are nestedness properties of the solution and subquadratic algorithms for the location of paths and subtrees. For the case of the median criterion, they prove that unlike the case where there is no existing facility, the continuous conditional median sub-tree problem is NP-hard and we develop a corresponding fully polynomial approximation algorithm. They also present subquadratic algorithms for almost all other models.

Wouter et al. (2011), contributed to the conditional location by writing; within research on world cities, much attention has been paid to Advanced Producer Services (APS) and their role within both global urban hierarchies and network formation between cities. What is largely ignored is that these APS provide services to firms operating in a range of different sectors. Does sector-specific specialization of advanced producer services influence the economic geography of corporate networks between cities? If so, what factors might explain this geographical pattern? This paper investigates these theoretical questions by empirically focusing on those advanced producer services related to the port and maritime sector. The empirical results show that the location of AMPS is correlated with maritime localisation economies, expressed in the presence of ship owners and the port-related industry as well as APS in general, but not by throughput flows of ports. Based on the findings, policy recommendations are addressed.

Berman and Simchi (2011), described an algorithm to solve conditional location problems (such as the conditional p-median problem or the conditional p-centre problem) on networks, where demand points are served by the closest facility whether existing or new. This algorithm requires the one-time solution of a (p + 1)-unconditional corresponding location problem using an appropriate shortest distance matrix.

Berman and Drezner (2007), discuss the conditional p-median and p-centre problems on a network. Demand nodes are served by the closest facility whether existing or new. Rather than creating a new location for an artificial facility and force the algorithm to locate a new facility thereby creating an artificial demand point, the distance matrix was just modified. They suggested solving both conditional problems by defining a modified shortest distance matrix.

Floyd-Warshall Algorithm

The Floyd-Warshall algorithm is an efficient matrix method algorithm to find all- pair shortest paths on a graph. That is, it is guaranteed to find the shortest path between every pair of vertices in a graph. The graph may have negative weight edges, but no negative weight cycles (for then the shortest path is undefined).

This algorithm can also be used to detect the presence of negative cycles; the graph has a negative cycles if at the end of the algorithm, the distance from a vertex v to itself is negative.

Algorithm

The Floyd-Warshall algorithm is an application of Dynamic Programming. Let $\mathrm{dist}(k, i, j)$ be the length of the shortest path from i and j that uses only the vertices 1,2,3,..........,k as intermediate vertices in N x N graph matrix . The following recurrence:

Step 1: if $k = 0$ is our base case, thus $\mathrm{dist}(0, i, j\ )$ is the length of the edge from vertex i to vertex jif it exists and infinite (∞) otherwise.

Step 2, using $\mathrm{dist}(0, i, j)$, it then computes $\mathrm{dist}(1, i, j)$ for all pairs of nodes i andj.

Step 3.using$\mathrm{dist}(1, i, j)$, it then computes $\mathrm{dist}(2, i, j)$, for all pairs of nodes i and j. It then repeats the process until it obtains $\mathrm{dist}(k, i, j)$ for all node pairs i andj when it terminates. The algorithm computes; $\mathrm{dist}(k, i, j) = \min\left(\mathrm{dist}(k-1, i, k) + \mathrm{dist}(k-1, k, j), \mathrm{dist}(k-1, i, j)\right)$; for any vertex i and vertexj, the length of the shortest path from i toj with all intermediate vertices $\leq k$ simply does not involve the vertex k at all (in which case it is the same as dist (k – 1, k ,j),or that the shorter path goes through vertex k, so the shortest path between vertex iand vertex j is the combination of the path from vertex i to k, and from vertex k to j.

After N iterations, there is no need any more to go through any more intermediate vertices, so the distance dist (N, i, j) represents the shortest distance between iand j.

Berman and Drezner's Algorithm

Berman and Drezner (2008), discussed a very simple algorithm that solves the conditional p-center problem on a network. The algorithm requires one-time solution of an unconditional p-center problem using an appropriate shortest distance matrix. Rather than creating a new location for an artificial facility and force the algorithm to locate a new facility there by creating an artificial demand point, they just modify the distance matrix.

Algorithm

Step 1: Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations.

Step 2: conditional problem is solved by defining a modified shortest distance matrix, from D to $\hat{D}$, where $\hat{D} = \min\left\{d_{ij}\ \min_{k \in Q}\{d_{ik}\}\right\} \forall i \in N, j \in C$ (Center). Even though D is symmetric but $\hat{D}$ is not symmetric.

Journal of Statistics and Actuarial Research
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The unconditional p-center problem using the appropriate $\hat{D}$ solves the conditional p-center problem. This is so since if the shortest distance from node ito the new p facilities are larger than $\min_{k\in Q}\{d_{ik}\}$, then the shortest distance to the existing facilities is been utilized. Notice that the size of $\hat{D}$ is $n \times |C|$ for the conditional p-center.

Step 3: Find the optimal new location using $\hat{D}$ for the network with the objective function

$$Min[G(x) = \max_{i=1,\ldots,n} \min\{d(X,i), d(Y,i)\}]$$

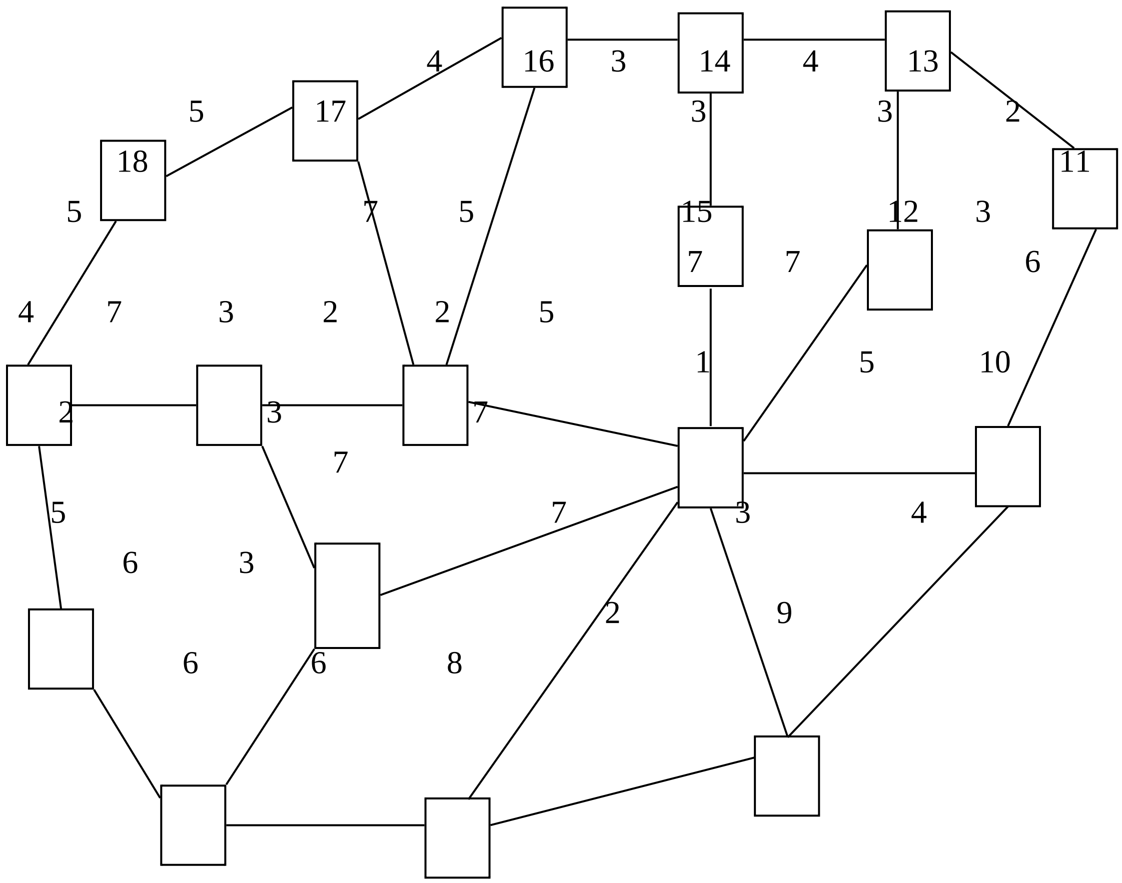

Figure 1: *Road Network of Berekum Municipality*.

Matrix Formation

Data Processing

The data of inter-towns and suburbs were entered manually into an edge distance matrix of size eighteen by eighteen using Microsoft excel. The edge distances of the nodes (suburbs) which are directly connected were allocated and nodes which were not directly connected have the edge distance entered as ‘inf’, representing infinite (∞) distance.

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A square matrix of size eighteen by eighteen is formed from figure 1(the road network) and the matrix is shown in Table 1.

Table 1: 18 × 18 Matrix input data for the Floyd-Warshall algorithm.

Nodes	1*	2	3	4*	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1*	0	5	inf	inf	inf	inf	7	7	3	5	7	inf	inf	inf	7	inf	inf	inf
2	5	0	2	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	5	7	inf
3	inf	2	0	7	inf	inf	3	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf
4*	inf	Inf	7	0	2	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	5
5	inf	Inf	inf	2	0	6	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf
6	inf	Inf	inf	inf	6	0	3	6	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf
7	7	Inf	3	inf	inf	3	0	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf
8	7	Inf	inf	inf	inf	6	inf	0	2	inf	inf	inf	inf	inf	inf	inf	inf	inf
9	3	Inf	inf	inf	inf	inf	inf	2	0	4	inf	inf	inf	inf	inf	inf	inf	inf
10	5	Inf	inf	inf	inf	inf	inf	inf	4	0	6	inf	inf	inf	inf	inf	inf	inf
11	7	Inf	inf	inf	inf	inf	inf	inf	inf	6	0	3	2	inf	inf	inf	inf	inf
12	inf	Inf	inf	inf	inf	inf	inf	inf	inf	inf	3	0	3	inf	2	inf	inf	inf
13	inf	Inf	inf	inf	inf	inf	Inf	inf	inf	inf	2	3	0	4	inf	inf	inf	inf
14	inf	Inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	4	0	3	3	inf	inf
15	7	Inf	Inf	inf	inf	inf	inf	inf	inf	inf	inf	2	inf	3	0	inf	inf	inf
16	inf	5	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	3	inf	0	4	inf
17	inf	7	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	4	0	5
18	inf	inf	inf	5	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	5	0

Note: Towns in Berekum Municipality and their codes:1 = Berekum, 2 = Jamdede, 3 = Domfete, 4=Jinijini, 5=Ayimon, 6=Benkasa, 7=Nsapor,8=Biadan,9=Senase,10=Kato,11=Kutre No. 2, 12 = Mpatapo,13=kutreNo.1,14=Akrofro,15=Mpatasie,16=Abisaase,17=Koraso,18=Fententaa.

*Currently, there are only two existing libraries; one at Berekum and the other one, community library at Jinijini. These communities form the set of existing facilities, thus node 1 and node 4 respectively.

Computation using Floyd-Warshall Algorithm

The 18 x 18 edge distance matrix was used as an input for the Floyd-Warshall algorithm coded in matlab.

Let $\mathrm{dist}(k, i, j)$ be the length of the shortest path from i and j that uses only the vertices 1,2,3,..........,k as intermediate vertices. The following recurrence:

Step 1; $k = 0$ is our base case, thus $\mathrm{dist}(0, i, j) = \mathrm{DA}(i, j)$ is the length of the edge from vertex i to vertex jif it exists and infinite (∞) otherwise.

Step 2; using $\mathrm{dist}(0, i, j)$, it then computes $\mathrm{dist}(1, i, j)$ for all pairs of nodes i andj.

Step 3; using $\mathrm{dist}(1, i, j)$, it then computes $\mathrm{dist}(2, i, j)$, for all pairs of nodes i and j**.** it then repeats the process until it obtains $\mathrm{dist}(k, i, j)$ for all node pairs i andj when it terminates. The algorithm computes the shortest paths as;

$\mathrm{dist}(k, i, j) = \min\big(\mathrm{dist}(k-1, i, j), \mathrm{dist}(k-1, i, k) + \mathrm{dist}(k-1, k, j)\big)$.

The calculated all pair shortest paths, Distance Matrix, D (in kilometres) obtained from the run of the Floyd-Warshall algorithm on Table 1 is summarised in Table 2.

In Table 2 column one and row one represent the demand nodes and potential location respectively; the other rows also represent the interconnecting road distances.

Journal of Statistics and Actuarial Research
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Table 4.3 All pair shortest paths Distance Matrix, DusingFloyd-Warshall algorithm.

Demand Nodes	Potential Locations																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	5	7	14	16	10	7	5	3	5	10	7	10	10	7	10	12	17
2	5	0	2	9	11	8	5	10	8	10	14	12	12	8	11	5	7	12
3	7	2	0	7	9	6	3	12	10	12	16	14	14	10	13	7	9	12
4	14	9	7	0	2	8	10	14	16	19	23	21	21	17	20	14	10	5
5	16	11	9	2	0	6	9	12	14	18	25	23	23	19	22	16	12	7
6	10	8	6	8	6	0	3	6	8	12	20	17	20	16	17	13	15	13
7	7	5	3	10	9	3	0	9	10	12	17	14	17	13	14	10	12	15
8	5	10	12	14	12	6	9	0	2	6	14	12	15	15	12	15	17	19
9	3	8	10	16	14	8	10	2	0	4	12	10	13	13	10	13	15	20
10	5	10	12	19	18	12	12	6	4	0	8	11	10	14	12	15	17	22
11	10	14	16	23	25	20	17	14	12	8	0	3	2	6	6	9	13	18
12	7	12	14	21	23	17	14	12	10	11	3	0	3	6	3	9	13	18
13	10	12	14	21	23	20	17	15	13	10	2	3	0	4	6	7	11	16
14	10	8	10	17	19	16	13	15	13	14	6	6	4	0	3	3	7	12
15	7	11	13	20	22	17	14	12	10	12	6	3	6	3	0	6	10	15
16	10	5	7	14	16	13	10	15	13	15	9	9	7	3	6	0	4	9
17	12	7	9	10	12	15	12	17	13	17	13	13	11	7	10	4	0	5
18	17	12	12	5	7	13	15	19	15	22	18	18	16	12	15	9	5	0

Berman and Drezner's Algorithm

At this point, we use the Berman and Drezner's algorithm (2008) to solve the problem. We begin by formulating the conditional p- center problem as

Journal of Statistics and Actuarial Research
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Vol 5, Issue 1, No.1, pp 1 - 20, 2021

$$Min[G(x) = \max_{i=1,\dots,n} \min\{d(X,i), d(Y,i)\}]$$

Let $d(x, y)$ be the shortest distance between any $x, y \in G$. Suppose that there is a set Q (|Q| = q) of existing facilities. Let $Y = (Y_1, Y_2, Y_3, \dots, Y_q)$ and $X = (X_1, X_2, X_3, \dots, X_p)$ be vectors of size q and p respectively, where Y_i is the location of existing facility and X_i is the location of new facility. Where $d(X, i)$ and $d(Y, i)$ is the shortest distance from the closest facility in X and Y respectively to the node i,(Berman and Simchi-Levi, 1990).

The set of location of new facilities $X = \{2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$ and the set of location of existing facilities $Y = \{1, 4\}$, then the conditional p – centre problem is to:

$$\text{Minimize } G(x) = \left[\max_{i=1,2,3,\dots,n} \min \left\{ \begin{array}{lll} d(2,i), & d(1,i), & d(4,i) \\ d(3,i), & d(1,i), & d(4,i) \\ d(5,i), & d(1,i), & d(4,i) \\ . & . & . \\ . & . & . \\ . & . & . \\ d(16,i), & d(1,i), & d(4,i) \\ d(17,i), & d(1,i), & d(4,i) \\ d(18,i), & d(1,i), & d(4,i) \end{array} \right\} \right]$$

Where $i = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

The Algorithm

Step 1: Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations.

Step 2: Conditional problem is solved by defining a modified shortest distance matrix, from D to $\hat{D}$, where $\hat{D} = \min\{d_{ij} \min_{k\in Q}\{d_{ij}\}\} \forall i \in N, j \in C$ (Center).

Even though D is symmetric but $\hat{D}$ is not symmetric.

Step 3: Find the optimal new location using $\hat{D}$ for the network with the objective function

$$Min[G(x) = \max_{i=1,\dots,n} \min\{d(X,i), d(Y,i)\}]$$

Computation Using Berman and Drezner's Algorithm.

A new shortest path distance matrix called modified shortest distance matrix, $\hat{D}$ is formed from Table 2 Thus from D to $\hat{D}$

Modified Shortest Distance Matrix, $\hat{D}$

By defining a modified shortest distance matrix $\hat{D}$ where

Journal of Statistics and Actuarial Research
ISSN 2518-881X
Vol 5, Issue 1, No.1, pp 1 - 20, 2021

$\hat{D}_{ij} = \min\{d_{ij} \min_{k \in Q}\{d_{ik}\}\} \forall i \in N, j \in C(\text{Center}).$

For node 1

$i = 1,\ j = 1 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{11} &= \min\{d_{11}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{0, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 2 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{12} &= \min\{d_{12}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{5, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 3 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{13} &= \min\{d_{13}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{7, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 4 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{14} &= \min\{d_{14}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{14, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 9 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{19} &= \min\{d_{19}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{3, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 10 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{1\ 10} &= \min\{d_{1\ 10}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{5, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 5 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{15} &= \min\{d_{15}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{16, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 6 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{16} &= \min\{d_{16}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{10, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 7 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{17} &= \min\{d_{17}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{7, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 8 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{18} &= \min\{d_{18}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{5, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 14 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{1\ 14} &= \min\{d_{1\ 14}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{10, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

$i = 1, j = 15 \text{ and } Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{1\ 15} &= \min\{d_{1\ 15}, \min\{d_{11}, d_{14}\}\} \\ &= \min\{7, \min\{0, 14\}\} \\ &= 0\end{aligned}$$

Journal of Statistics and Actuarial Research
ISSN 2518-881X
Vol 5, Issue 1, No.1, pp 1 - 20, 2021

$i = 1,\ j = 11$ and $Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{1\ 11} &= \min\{d_{1\ 11}, \min\{d_{11}, d_{14}\}\}\\ &= \min\{10, \min\{0,14\}\}\\ &= 0\end{aligned}$$

$i = 1,\ j = 12$ and $Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{1\ 12} &= \min\{d_{1\ 12}, \min\{d_{11}, d_{14}\}\}\\ &= \min\{7, \min\{0,14\}\}\\ &= 0\end{aligned}$$

$i = 1,\ j = 13$ and $Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{1\ 13} &= \min\{d_{1\ 13}, \min\{d_{11}, d_{14}\}\}\\ &= \min\{10, \min\{0,14\}\}\\ &= 0\end{aligned}$$

$i = 1,\ j = 16$ and $Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{1\ 16} &= \min\{d_{1\ 16}, \min\{d_{11}, d_{14}\}\}\\ &= \min\{10, \min\{0,14\}\}\\ &= 0\end{aligned}$$

$i = 1,\ j = 17$ and $Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{1\ 17} &= \min\{d_{1\ 17}, \min\{d_{11}, d_{14}\}\}\\ &= \min\{12, \min\{0,14\}\}\\ &= 0\end{aligned}$$

$i = 1,\ j = 18$ and $Q = \{1, 4\}$

$$\begin{aligned}\hat{D}_{1\ 18} &= \min\{d_{1\ 18}, \min\{d_{11}, d_{14}\}\}\\ &= \min\{17, \min\{0,14\}\}\\ &= 0\end{aligned}$$

Therefore, the modified shortest distance for node 1 at i = 1, j = 1 is 0, i = 1, j = 2 is 0 and that of i = 1, j = 3 is also 0.

Table 3 then summarizes the results of $\hat{D}$ into a modified shortest distance matrix. Column one representing demand nodes and all other columns representing the minimum interconnecting distance when demand nodes are compared with existing facilities nodes. Details of modified shortest distance is elaborated in appendix B

Journal of Statistics and Actuarial Research
ISSN 2518-881X
Vol 5, Issue 1, No.1, pp 1 - 20, 2021

Table 3: Modified Shortest Distance Matrix, $\hat{D}$

Demand Nodes	Potential location																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	5	0	2	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
3	7	2	0	7	7	6	3	7	7	7	7	7	7	7	7	7	7	7
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	2	2	2	2	0	2	2	2	2	2	2	2	2	2	2	2	2	2
6	8	8	6	8	6	0	3	6	8	8	8	8	8	8	8	8	8	8
7	7	5	3	7	7	3	0	7	7	7	7	7	7	7	7	7	7	7
8	5	5	5	5	5	5	5	0	2	5	5	5	5	5	5	5	5	5
9	3	3	3	3	3	3	3	2	0	3	3	3	3	3	3	3	3	3
10	5	5	5	5	5	5	5	5	4	0	5	5	5	5	5	5	5	5
11	10	10	10	10	10	10	10	10	10	8	0	3	2	6	6	9	10	10
12	7	7	7	7	7	7	7	7	7	7	3	0	3	6	3	7	7	7
13	10	10	10	10	10	10	10	10	10	10	2	3	0	4	6	7	10	10
14	10	8	10	10	10	10	10	10	10	10	6	6	4	0	3	3	7	10
15	7	7	7	7	7	7	7	7	7	7	6	3	6	3	0	6	7	7
16	10	5	7	10	10	10	10	10	10	10	9	9	7	3	6	0	4	9
17	10	7	9	10	10	10	10	10	10	10	10	10	10	7	10	4	0	5
18	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0

Comparing various road distances with the existing nodes 1 and 4, the minimum distance is always zero. Hence, in the next Table, the set of demand nodes and potential location of the existing facilities are removed from the modified shortest path distance matrix, $\hat{D}$ in Table 3.

Journal of Statistics and Actuarial Research
ISSN 2518-881X
Vol 5, Issue 1, No.1, pp 1 - 20, 2021

www.iprjb.org

Table 4: Modified Distance Matrix with Nodes 1 and 4 Removed.

Demand Nodes	Potential Location															
	2	3	5	6	7	8	9	10	11	12	13	14	15	16	17	18
2	0	2	5	5	5	5	5	5	5	5	5	5	5	5	5	5
3	2	0	7	6	3	7	7	7	7	7	7	7	7	7	7	7
5	2	2	0	2	2	2	2	2	2	2	2	2	2	2	2	2
6	8	6	6	0	3	6	8	8	8	8	8	8	8	8	8	8
7	5	3	7	3	0	7	7	7	7	7	7	7	7	7	7	7
8	5	5	5	5	5	0	2	5	5	5	5	5	5	5	5	5
9	3	3	3	3	3	2	0	3	3	3	3	3	3	3	3	3
10	5	5	5	5	5	5	4	0	5	5	5	5	5	5	5	5
11	10	10	10	10	10	10	10	8	0	3	2	6	6	9	10	10
12	7	7	7	7	7	7	7	7	3	0	3	6	3	7	7	7
13	10	10	10	10	10	10	10	10	2	3	0	4	6	7	10	10
14	8	10	10	10	10	10	10	10	6	6	4	0	3	3	7	10
15	7	7	7	7	7	7	7	7	6	3	6	3	0	6	7	7
16	5	7	10	10	10	10	10	10	9	9	7	3	6	0	4	9
17	7	9	10	10	10	10	10	10	10	10	10	7	10	4	0	5
18	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0

Using Table 4 above, the optimal new location is found using the objective function;

$$Min[G(x) = \max_{i=1,\ldots,n} \min\{d(X,i), d(Y,i)\}]$$

Journal of Statistics and Actuarial Research
ISSN 2518-881X
Vol 5, Issue 1, No.1, pp 1 - 20, 2021
www.iprjb.org

With $Y = \{1,4\}$ and $X = \{2,3,5,6,7,8,9,10,11,12,13,14,15,16,17,18\}$

For X = 2

$i = 1,$
$\min\{d(2,1), d(1,1), d(4,1)\}$
$= \min\{0,0,0\}$
$= 0$

$i = 2,$
$\min\{d(2,2), d(1,2), d(4,2)\}$
$= \min\{0,5,5\}$
$= 0$

$i = 3,$
$\min\{d(2,3), d(1,3), d(4,3)\}$
$= \min\{2,7,7\}$
$= 2$

$i = 4,$
$\min\{d(2,4), d(1,4), d(4,4)\}$
$= \min\{0,0,0\}$
$= 0$

$i = 5,$
$\min\{d(2,5), d(1,5), d(4,5)\}$
$= \min\{2,2,2\}$
$= 2$

$i = 6,$
$\min\{d(2,6), d(1,6), d(4,6)\}$
$= \min\{8,8,8\}$
$= 8$

$i = 7,$
$\min\{d(2,7), d(1,7), d(4,7)\}$
$= \min\{5,7,7\}$
$= 5$

$i = 8,$
$\min\{d(2,8), d(1,8), d(4,8)\}$
$= \min\{5,5,5\}$
$= 5$

$i = 9,$
$\min\{d(2,9), d(1,9), d(4,9)\}$
$= \min\{3,3,3\}$
$= 3$

$i = 10,$
$\min\{d(2,10), d(1,10), d(4,10)\}$
$= \min\{5,5,5\}$
$= 5$

$i = 11,$
$\min\{d(2,11), d(1,11), d(4,11)\}$
$= \min\{10,10,10\}$
$= 10$

$i = 12,$
$\min\{d(2,12), d(1,12), d(4,12)\}$
$= \min\{7,7,7\}$
$= 7$

$i = 13,$
$\min\{d(2,13), d(1,13), d(4,13)\}$
$= \min\{10,10,10\}$
$= 10$

$i = 16,$
$\min\{d(2,16), d(1,16), d(4,16)\}$
$= \min\{5,10,10\}$
$= 5$

Journal of Statistics and Actuarial Research
ISSN 2518-881X
Vol 5, Issue 1, No.1, pp 1 - 20, 2021

$i = 14,$
$\min\{d(2,14), d(1,14), d(4,14)\}$
$= \min\{8,10,10\}$
$= 8$
$i = 15,$
$\min\{d(2,15), d(1,15), d(4,15)\}$
$= \min\{7,7,7\}$
$= 7$

$i = 17,$
$\min\{d(2,17), d(1,17), d(4,17)\}$
$= \min\{7,10,10\}$
$= 7$
$i = 18,$
$\min\{d(2,18), d(1,18), d(4,18)\}$
$= \min\{5,5,5\}$
$= 5$

Therefore, at X =2, the maximum is 10, the maximum at X =3 is 10 and that of X =5 is 10.The results are then summarized in Table 5 with column 18 representing the maximum distance on each row. Details of the calculation are presented in appendix C.

Table 5: Optimal LocationMin$(\mathbf{g}(\mathbf{x}))$ using $\hat{D}$

Demand Nodes	Potential Location															
	2	3	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Maximum
2	0	2	2	8	5	5	3	5	10	7	10	8	7	5	7	5	10
3	2	0	2	6	3	5	3	5	10	7	10	10	7	7	9	5	10
5	5	7	0	6	7	5	3	5	10	7	10	10	7	10	10	5	10
6	5	6	2	0	3	5	3	5	10	7	10	10	7	10	10	5	10
7	5	3	2	3	0	5	3	5	10	7	10	10	7	10	10	5	10
8	5	7	2	6	7	5	2	5	10	7	10	10	7	10	10	5	10
9	5	7	2	8	7	2	0	4	10	7	10	10	7	10	10	5	10
10	5	7	2	8	7	5	3	0	8	7	10	10	7	10	10	5	10
11	5	7	2	8	7	5	3	5	0	3	2	6	6	9	10	5	10
12	5	7	2	8	7	5	3	5	3	0	3	6	3	9	10	5	10
13	5	7	2	8	7	5	3	5	2	3	0	4	6	7	10	5	10
14	5	7	2	8	7	5	3	5	6	6	4	0	3	3	7	5	8
15	5	7	2	8	7	5	3	5	6	3	6	3	0	6	10	5	10

16	5	7	2	8	7	5	3	5	9	7	7	3	6	0	4	5	9
17	5	7	2	8	7	5	3	5	10	7	10	7	7	4	0	5	10
18	5	7	2	8	7	5	3	5	10	7	10	10	7	9	5	0	10
Minimum																	8

Discussion of Results

A location problem is considered as a conditional p-centre problem when given the location of existing facilities and we are located p-additional facility or facilities to minimize the maximum travel distance between the demand points, each to its nearest facility whether *existing or new***.** Berman and Simchi-Levi showed how to solve the conditional p-median and p-centre problems by solving a single unconditional (p+1)-centre problem. Chen and Handler presented a relaxation algorithm for solving the conditional p-centre problem. Drezner showed how the conditional p-centre problem can be solved by solving O (log n) p-centre problems.

In this current paper, the researchers present a new algorithm for solving the conditional p-centre problem which relies on Floyd-Warshall's algorithm as well as algorithm given by Berman and Drezner. With the algorithm from Berman and Drezner, considering the eighteen node network depicted in Figure 4.1 and solving the conditional p-centre problem with the existing facilities at node 1 and node 4 (i.e Berekum and Jinijini respectively), Table 2, Table 3 and Table 4 are the all pair shortest paths distance matrices, the modified shortest distance matrix and the modified shortest distance matrix with nodes 1 and 4 removed respectively.Table 5 indicates that using Berman and Drezner (2008) algorithm on the modified shortest distance matrix, the optimal new location is at node 14 (Akrofro). Therefore the proper place to site the new facility is Akrofro among all the other towns with an objective function value of 8.

Anyabilla (2014) used Berman and Drezner to locate three additional warehouses for Uniliver Ghana Limited as a p-centre problem given that some existed warehouses are already located in the Ashanti Region of Ghana. Similarly, Kwarteng (2012) also used Berman and Drezner to locate additional Hospital facility in Antoakrom in the Amansie West District in the Ashanti Region.

Conclusion

Floyd-Warshall's algorithm was used to find all pair shortest path between nodes using figure 1 which resulted in eighteen by eighteen (1818) matrix in Table 1.

The formulation of the model for the location of an additional library facility in the Berekum Municipality is indicated on page 10. The model was solved using Berman and Drezner's algorithm as shown on page 11.

Considering the objective function from Berman and Drezner the library facility from Table 5 should be located at Akrofro.

The minimum objective function value obtained was 8 kilometres which mean that the minimum distance travelled by the farthest library user to the new library facility at Akrofro is 8 kilometres.

Recommendations

Considering the conclusion above, researchers recommend that Berekum Municipal Assembly, Municipal Education Directorate, Non-Governmental Organizations (NGOs) and individuals who would like to locate an additional library facility in Berekum Municipality should locate it at Akrofro.

In addition to the above, it is recommended that stakeholders should adopt the use of Berman and Drezner's algorithms in establishing facilities such as markets, hospitals, recreational centres and so on.

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