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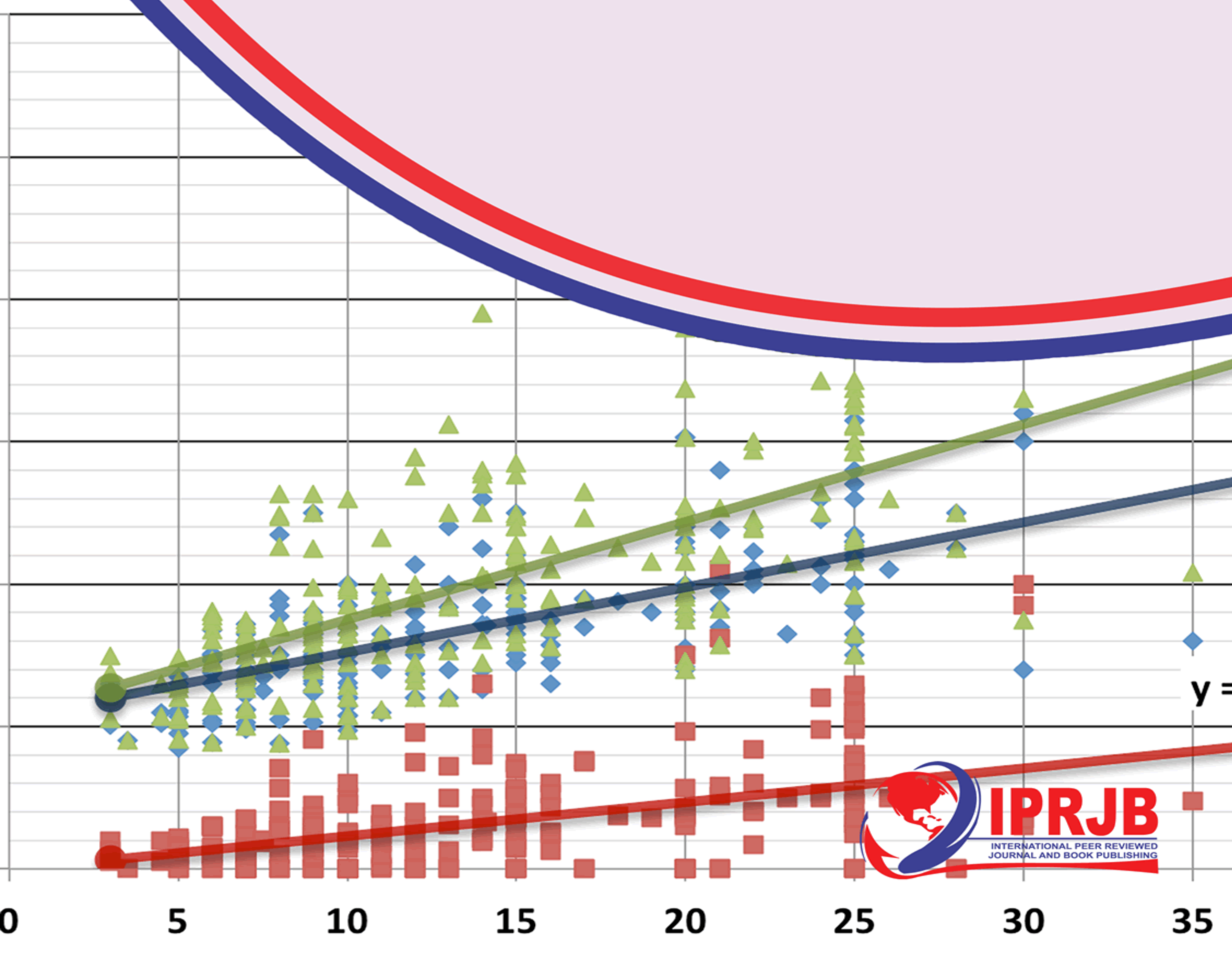

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MATHEMATICAL APPLICATION OF REVISED SIMPLEX MODELLING IN OPTIMAL LOAN PORTFOLIO

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Abstract

Purpose: The main aim of this study is to develop Linear Programming (LP) model to help ADB, Sunyani branch in the Brong Ahafo Region to allocate their funds to prospective loan seekers in order to maximize profits.

Methodology: To achieve this aim, a secondary data were extracted from the annual reports and financial statements of the bank. Based on these empirical data, LP model was formulated. A computerized software application called LP Solver based on Revised Simplex Algorithm was used to solve the problem.

Results: The results from the model showed that ADB, Sunyani branch would be making annual profit of GH¢ 476732.00 on loans alone as against GH¢ 190693.00 in 2011 if they stick to the model.

Unique contribution to Theory, Practice and Policy: From the study, it was realized that the Scientific method used to develop the proposed model can have a significant increase in the Bank's profit margin if put into use.

Keywords: Application, Linear Programming (LP), Portfolio, Optimal

1.0 INTRODUCTION

Lending is one of the main activities of banks in Ghana and other parts of the world. This is evidenced by the volume of loans that constitute banks assets and the annual substantial increase in the amount of credit granted to borrowers in the private and public sectors of the economy. According to Comptroller (1998), lending is the principal business for most commercial banks. Loan portfolio is therefore typically the largest asset and the largest source of revenue for banks.

In view of the significant contribution of loans to the financial health of banks through interest income earnings, these assets are considered the most valuable assets of banks. A survey in 2006 on the Ghanaian banking sector revealed that loans accounted for about fifty percent (50%) of total bank assets which had increased from forty one point five percent (41.5%) in 2005 (Appertey & Arkaifie, 2006, as cited by Infodata Associates, 2009). Also, Infodata Associates, (2009) observed in 2007 that the figure increased to fifty three percent (53%) of the industry's total assets of GH¢ 7,795.6 million.

The reason why banks give much attention to the lending activity, especially in periods of a stable economic environment, is that a substantial amount of banks income is earned on loans which contribute significantly to the financial performance of banks. A financial report of ADB in 2007, indicated that out of the total interest income of GH¢42,327,367.00 earned in that year, about sixty six point five percent (66.5%) was earned on loans and advances.

From the forgoing, healthy loan portfolios are vital assets for banks in view of their positive impact on the performance of banks. Unfortunately, some of these loans usually do not perform and eventually result in bad debts which affect banks earnings on such loans. These bad loans become cost to banks in terms of their implications on the quality of their assets portfolio and profitability. This is because in accordance with banking regulations, banks make provisions for non-performing loans and charge for bad loans which reduce their loan portfolio and income. For example in February, 2009, Bank of Ghana report revealed that non-performing loans ratio increased from six point four percent (6.4%) in 2007, to seven point seven percent (7.7%) in 2008. A cursory study of the annual reports and financial statements of banks in Ghana indicate that bad loans are seriously affecting most banks hence necessitating a study into the problem. (ADB, 2008).

1.2 Statement of the Problem

Agricultural development banks record marginal profits with some running at a loss. Loan is granted in a form of contract between the bank and the borrower. Due to poor allocation and loans disbursement, the banks are not able to optimize profits from the loans. Monies that could have been used to offer social services like building of schools, hospitals, markets etc in the community in which they operate goes into "Bad Debts". As a result, the ADB is advocating for a device or a model that would enable the bank to solve the problem of loan disbursement optimally for both long term and short term basis in other to maximize their profit.

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It is from the stated observations that this research work is skewed at developing a linear programming model with the specific purpose of providing an optimal solution of banks problem with a case study of ADB in Sunyani.

1.3 Objectives

1.3.1 General Objectives

The general objectives of the study were to investigate how mathematical Linear Programming Model (LP Solver) would help to disburse loan to customers effectively and efficiently to maximize profit. Also, the study would investigate how Linear Programming Model application in banking would help decision makers to formulate prudent and effective loan policies.

1.3.2 Specific Objectives

1. To what extent would LP Model help in disbursing funds for loan effectively and efficiently in order to maximize profit?

2. How would the results of the LP Model help policy makers to formulate prudent and effective loan policies?

2.0 LITERATURE REVIEW

2.1 Modern Portfolio Theory

According to Bodie, Kane and Alan (2009), the concept of investment diversification is an old one and existed long before modern finance theory. It was, however, not until 1952 that Harry Markowitz published a formal model of portfolio selection based on diversification principles. This work contributed to Markowitz receiving the Nobel Prize in Economics in 1990. His model can be regarded as the first step in portfolio management, which is the identification of the efficient set of portfolios or the efficient frontier of risky assets.

Actually, the work began in 1900 when the French mathematician, Louis Bachelier, studied financial markets. Based on his studies, Bachelier argued that prices will go up or down with equal probability and that their volatility is measurable. The so-called bell curve was born, whereby the distribution of price movements is thought to be bell-shaped with very large changes assumed to be extremely rare. It was Markowitz who took the first step in applying Bachelier's ideas (Mandelbrot, 2004).

2.2 Markowitz's Portfolio Theory

In the 1950s, the investment community talked about risk but there was no measurable specification for the term. However, investors were eager to quantify their risk variable. Markowitz showed that the variance of the rate of return was an important measure of risk under a reasonable set of assumptions and came forward with the formulas for computing the variance of the portfolio. When Markowitz first published his ideas of portfolio selection in 1952, he rejected the notion that investors should maximize discounted returns and choose their portfolio accordingly.

Markowitz's view was that this rule failed to imply diversification, no matter how the anticipated returns were formed. The rule he rejected implied that the investor should place all of his or her funds in the security with the greatest discounted value. He also rejected the law of large numbers in portfolios made up of securities, objecting to the claim that it would result in both maximum expected returns and minimum variance, and pointing out that returns from securities are too inter-correlated for all variance to be eliminated with diversification.

Markowitz also pointed out that a portfolio with maximum expected returns is not necessarily the one with the minimum variance. Hence, there should be a rate at which the investor can gain expected returns by accepting more variance, or reduce variance by giving up expected returns. Building on these observations he presented the expected returns-variance of return rule (Markowitz, 1952). Markowitz's idea was that investors should hold mean-variance efficient portfolios. While not an entirely new concept, mean-variance optimization was not a widely used strategy at the time. Most investment managers were focusing their efforts on identifying securities with high expected returns (Chan, Karceski, & Lakonishok, 1999).

These authors Chan, Karceski, and Lakonishok, (1999) formally presented Markowitz (1952) view that although investors want to maximize returns on securities they also want to minimize uncertainty, or risk. These are conflicting objectives which must be balanced against each other when the investor makes his or her decision. Markowitz asserts that investors should base their portfolio decisions only on expected returns, i.e. the measure of potential rewards in any portfolio, and standard deviation, the measure of risk. The investor should estimate the expected returns and standard deviation of each portfolio and then choose the best one on the grounds of the relative magnitudes of these two parameters (Sharpe, Alexander & Bailey, 1999).

As previously mentioned, Markowitz rejected the expected returns rule on the grounds that it neither acknowledged nor accounted for the need for diversification, contrary to his expected return-variance of return rule. In addition, he concluded that the expected return variance of return rule not only revealed the benefits of diversification but that it pointed towards the right type of diversification for the right reason. It is not enough to diversify by simply increasing the number of securities held. If, for example, most of the firms in the portfolio are within the same industry they are more likely to do poorly at the same time than firms in separate industries.

In the same way it is not enough to make variance small to invest in large number of securities. It should be avoided to invest in securities with high covariance among themselves and it is obvious that firms in different industries have lower covariance than firms within the same industry Markowitz, (1952), simply put, Markowitz concluded that by mixing stocks that flip tail and those that flip heads you can lower the risk of your overall portfolio. If you spread your investments across unrelated stocks you will maximize your potential profit whether the economy is slowing down or growing. If you then add more and more stock in different combinations you have what Markowitz called an efficient portfolio. An efficient portfolio is the portfolio which gives the highest profit with the least risk.

The aim of Markowitz's methods is to construct that kind of portfolio (Mandelbrot, 2004). Until Markowitz, (1952), suggested this approach to portfolio analysis no full and specific basis existed to justify diversification in portfolio selection. Also the concept of risk had rarely been defined in a thorough manner in portfolio analysis before Markowitz's writings, let alone treated analytically. With his approach these issues, diversification and risk, got a specified framework and a workable algorithm for employing that framework for practical problems was provided. Markowitz did not, however, suggest a preferred technique for security analysis or a suitable method for portfolio selection. He concentrated on providing a general structure for the whole process and providing an algorithm for performing the task of portfolio analysis (Sharpe, 1967).

Markowitz created a theory of portfolio choice in the uncertain future. He quantified the difference between the risk that was taken on individual assets and the aggregated risk of the portfolio. He showed that the portfolio risk came from covariance of the assets which made up the portfolio. The marginal contribution of a security to the portfolio return variance is therefore measured by the covariance between the return of the security and the return of the portfolio but not by the variance of the security itself. In his writings, Markowitz argues that the risk of a portfolio is less than the risk of each asset in the portfolio taken individually and provides quantitative evidence of the merits of diversification (Amenc & Sourd, 2003).

In his model of portfolio management, Markowitz identified the efficient set of portfolios, or the efficient frontier of risky assets. The principal idea behind the frontier set of risky portfolios is that the investor should only be interested in the portfolio which gives the highest expected return for any given risk level. Also the frontier is a set of portfolios that minimizes the variance for any target expected return (Bodie, Kane & Marcus, 2009).

2.3 Linear Programming Model

Linear programming theory and technique have been successfully applied to various transportation problems almost since its early beginning. A famous example is given by Dantzig to adapt his simplex method to solve Hitchcock`s transportation problem. The terminology, such as transportation and assignment problems, and have become a standard in these contexts since then. Linear programming methods were first used to study origin-destination distributions in 1970s.

2.4 Linear Programming in Financial Management

The use of linear and other types of mathematical programming techniques have received coverage in the extensive banking literature,(Chambers & Chames ,1961), as well as (Cohen & Hammer 1967;1972), developed a series of sophisticated linear programming models for managing the balance sheet of larger banks. While, Waterman and Gee (1963) proposed less elegant formulations which were better suited for the small to medium-sized bank. Several programming models have also been proposed for managing a bank's investment security portfolio, including those by, (Booth, 1972).

2.5 Linear Programming for Bank Portfolio Management

Various portfolio theories have been propounded for the management of bank funds. In 1961, Ronald Robinson reserved the proposed four priorities of the use of banks funds. These include primary reserves, (or protective investment), loans and advances (customer credit demand) and investment account (open market investment for income) in descending order of priority. His assessment has been fully supported in other works by (Sheng-Yi & Yong,1988). A bank has to place primary reserves at the top of the priority in order to comply with the minimum legal requirement, to meet any immediate withdrawal demand by depositors and to provide means of clearing cheque and credit obligations among banks.

Secondary reserves include cash items from banks, treasury bills and other short-term securities. Bank should have to satisfy customers' loan demand before allocating the balance of the funds in the investment market. Loans and investment are in fact complementary. According to Robinson, (1961) investment should be tailored to the strength, seasonality and character of loan demand. He reiterated that banks that experience sharp seasonal fluctuations in loan demand need to maintain more liquidity in their investment programmed. Moreover, during a boom when loan demand is high and creditworthy customers are available, banks should allocate more funds to loans and fewer funds to investment, and vice versa during recession when loan demand is low. According to Robinson, (1961) the crucial banking problem is to resolve the conflict between safety and profitability in the employment of bank funds. The conflict is essentially the problem between liquidity and the size of the earning assets. Robinson suggested that where there is a conflict between safety and profitability, it is better to err on the side of safety.

The best practice is identifying procedures that can bring out the optimal mixture of management of banks funds. According to Tobin (1965) portfolio theory can be applied to bank portfolio management in that a bank would maximize the rates of return of its portfolio of assets, subject to the expected degree of risk and liquidity. Chambers and Charnes (1961) applied linear programming analysis on the consolidated balance sheets of commercial banks in Singapore for the period 1978-1983. The results show that large banks do not try to maximize the returns of their portfolios, subject to legal, policy, bounding and total assets constraints, which denote riskiness and liquidity of the portfolio of assets. In a direct way, banks conform to the portfolio choice theory; they have to balance yield and liquidity against security. They pointed out that although the computer cannot replace a manager, linear programming can serve as a useful guide.

3.0 METHODOLOGY

The data for this study consist of secondary data collected from ADB, Sunyani branch in the Brong Ahafo Region of Ghana annual report, journals and dailies. In order for the bank to maximize profit, the proposed model would be based strictly on the Bank's loan policy and its previous history on loan disbursement. The model would be solved using revised Simplex Algorithm.

The computerized software application program called LP Solver based on the revised Simplex Algorithm would be used to facilitate the solution of the linear programming model developed. The LP solver is considered the best option for the project because the spreadsheet offers a very convenient data entry and editing features which allows for a greater understanding of how to construct linear programs. The method would also be selected due to the fact that it is a popular program used by the operational researchers. The model would be analyzed by the use of LP Solver

3.1 The Revised Simplex Method

The original Simplex method is a straight forward algebraic procedure. However, this way of executing the algorithm (in either algebraic or tabular form) is not the most efficient computational procedure for computers because it computes and stores many numbers that are

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not needed at the current iteration and that may not even become relevant for decision making at subsequent iterations. The only pieces of information relevant at each iteration are:

(i) The coefficients of the non-basic variables.

(ii) The coefficients of the entering basic variable in the other equations.

(iii)The right–hand sides of the equations.

It would be very useful to have a procedure that could obtain this information efficiently without computing and storing the other coefficients. These considerations motivated the development of the revised simplex method. This method was designed to accomplish exactly the same things as the original simplex method, but in a way that is more efficient for execution on computer. Thus, it is a streamlined version of the original procedure. It computes and stores only the information that is currently needed, and it carries along the essential data in a more compact form.

The revised simplex method explicitly uses matrix manipulations, so it necessary to describe the problem in matrix notation. Using matrices, our standard form for the general linear programming model becomes

Maximize Z = c x,

Subject to

$Ax \leq b$ and $x \geq 0,$

Where c is the row vector

$C = [C_1, C_2, C_3, \ldots, C_n]$

x, b, are the column vectors such that

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{pmatrix}, \qquad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ . \\ . \\ b_n \end{pmatrix} \text{ and } A \text{ is matrix with } A = \begin{pmatrix} a_{11} & a_{12}\ldots & a_{1n} \\ a_{21} & a_{22}\ldots & a_{2n} \\ a_{m1} & a_{m2}\ldots & a_{mn} \end{pmatrix}$$

To obtain the augmented form of the problem, introduce the column vector of slack variables.

$$\begin{pmatrix} x_{n+1} \\ x_{n+2} \\ . \\ . \\ . \\ x_{n+m} \end{pmatrix}$$ so that the constraints become $[A\ I]\begin{pmatrix} x \\ x_s \end{pmatrix} = b$ and $x = \begin{pmatrix} x \\ x_s \end{pmatrix} \geq 0$ where I is the $m \times n$ identity matrix.

3.2 Solving For a Basic Feasible Solution

Recall that the approach of the Simplex method is to obtain a sequence of improving Basic Feasible solutions until on optimal solution is reached. One of the key features of the

Revised Simplex method involves the way in which it solves for each new Basic Feasible solution after identifying its basic and non-basic. Given the variables, the resulting basic solution is the solution of the *m* equations.

[A I] $\begin{pmatrix} x \\ x_s \end{pmatrix} = b$, in which the non-basic variables from the $n+m$ elements of $\begin{pmatrix} x \\ x_s \end{pmatrix}$ are set equal to zero.

Eliminating these n variables by equating them to zero leaves a set of m equations in *m* unknowns (the basic variables). This set of equations can be denoted by $BX_B = b$, where the vector of basic variables

$x_B = \begin{pmatrix} x_{B1} \\ x_{B2} \\ x_{Bm} \end{pmatrix}$ is obtained by eliminating the non-basic variables from $\begin{pmatrix} x \\ x_s \end{pmatrix}$, and the basis matrix

$$A = \begin{pmatrix} B_{11} & B_{12}\cdots & B_{1m} \\ B_{21} & B_{22}\cdots & B_{2m} \\ \cdots & \cdots & \cdots \\ B_{m1} & B_{m2} & B_{mm} \end{pmatrix}$$

is obtained by eliminating the columns corresponding to coefficients of non-basic variables from [A I]. (in addition, the elements of x B and, therefore, the columns of B may be placed in a different order when the Simplex method is executed). The Simplex method introduces only basic variables such that B is nonsingular, so that B always will exist. Therefore, to solve

$B\,X_B = b$, both sides are pre multiplied by B^{-1}

$BX_{B} = B^{-1}b$

Since $B^{-1}B = I$, the desired solution for the basic variables *is* $X_B = B^{-1}\, b$. Let C_B be vector whose elements the objective function coefficients (including Zeros for slack variables) are for the corresponding elements of X_B. The value of the objective function for this basic solution is then $Z = C_B X_B = C_B B^{-1} b$

3.3 Revised Simplex Algorithm

Original simplex method calculates and stores all numbers in the tableau-many are not needed.

Revised Simplex Method (more efficient for computing). Used in all commercial available package. (e. g. IBM MPSX, CDC APEX III)

Max $Z = cx$

Subject to; $Ax \leq b$

$X \geq 0$

$[A \ \ I]\begin{pmatrix} x \\ x_s \end{pmatrix} = (b)$

$X_s =$ slack variables Basis matrix: columns relating to basic variables

$$B = \begin{pmatrix} B_{11} & .... & B_{1m} \\ ... & ... & ... \\ B_{m1} & ... & B_{mm} \end{pmatrix} \qquad \text{(Initially } B = I\text{)}$$

Basic variable values: $x_B = \begin{pmatrix} x_{B1} \\ ... \\ x_{Bm} \end{pmatrix}$

At any iteration non – basic variable = 0

$Bx_B = b$. Therefore $x_B = B^{-1}b, B^{-1} \rightarrow$ inverse matrix. At any iteration, given the original vector and the

Inverse matrix, X_B (current R.H.S.) can be calculated. $Z = C_B x_B$ where $C_B =$ objective coefficient of basic variables.

3.4 Steps in the Revised Simplex Method.

1. Determine entering variable, X_J with associated vector P_J

(i) Determine the coefficient of the basic variable, C_B

(ii) Compute $Y = C_B B^{-1}$

(iii) Compute $z_j - cj = YP_j - c_j$ for all non-basic variables.

(iv) Choose largest negative value (maximization) if none, stop.

2. Determine leaving variable, X_r, with associated vector Pr.

(i) Compute $X_B = B^{-1}b$ (current R.H.S.)

(ii) Compute current constraint coefficients of entering variable:

$a^j = B^{-1}P_j$

X_r is associated with

$\theta = \text{Min } \{(x_B)_k / a^j k, a^j{}_k > 0\}$

(Minimum ratio rule)

3. Determine next basis i.e. calculate B^{-1}

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Go to step 1.

3.5 Duality

Corresponding to any given linear programming problem called the primal problem, is another linear programming problem called the Dual Problem. Since a given linear programming problem can be stated in several forms (standard form, canonical form, etc), it follows that the forms of the dual problem will depend on the form of the primal problem. A fundamental of the primal dual-relationship is that the optimal solution to either the primal or the dual problem also provides optimal solution to the other.

A maximization problems with all the less-than or equal to constraint and the non- negative requirement for the decision variables is said to be in canonical form as in example 3.3 used below. If the dual problem has optimal solution, then the primal also has an optimal solution and vice versa. The values of the optimal solution to the dual and primal are equal. These are rules for converting the primal problem in any form into its dual.

Table 3.1: Converting Primal Problem to Dual Form.

PRIMAL PROBLEM	DUAL PROBLEM
Maximization	Minimization
Coefficient of objective function	Right hand sides of constraint
Coefficient of i^{th} constraint	Coefficient of i^{th} variable
i^{th} constraint is an inequality of the form $\leq$	i^{th} variable satisfies ≥ 0
i^{th} constraint is an inequality	i^{th} variable is unrestricted
i^{th} variable is unrestricted	i^{th} constraint is an equality
i^{th} variable satisfies ≥ 0	i^{th} constraint is an inequality of the form $\geq$
Number of variable	Number of Constraint
Number of Constraint	Number of variable

3.6 Degeneracy

A linear program is said to be degenerate if one or more basic variables have a value zero. This occurs whenever there is a tie in the minimum ratio prior to reaching the optimal solution. This may result in cycling, that is the procedure could possibly alternate between the same set of no optimal basic feasible solutions and never reach the optimal solution.

In order to overcome this problem, the following steps may be used to break the tie between the key row tie,

(i) Select the rows where the ties are found for determining the key row.

(ii) Find the coefficient of the slack variable and divide each coefficient by the coefficients in the key column in order to break the tie. If the ratios at this stage do not break the tie, find the similar ratios for the coefficient of the decision variables.

(iii) Compare the resulting ratio column by column.

(iv)Select the row which has the smallest ratio and this now becomes the key row.

3.7 Sensitivity Analysis

Suppose that you have just completed a linear programming solution which has a major impact.

How much would the result change if your basic data is slightly wrong? Would that have a minor impact on your result? Would it give a completely different outcome, or change the outcome only slightly? These are the kind of questions addressed by sensitivity analysis. It allows us to observe the effect of changes in the parameters in the LP problem on the optimal solution. It is also useful when the values of the problem parameters are not known. Formally, the question is this; is my optimum solution sensitive to a small change in one of the original problem coefficient. This sort of examination of impact of the input data on output results is very crucial.

The procedure and algorithm of mathematical programming are important, but the problems that really appear in practice are usually associated with data: getting it all, and getting accurate data. What is required in sensitivity analysis is which data has significant impact on your results. There are several ways to approach sensitivity analysis. If your model is small enough to solve quite quickly, you can simply change the initial data and solve the model again to see what results you get. At the extreme, if your model is very large and takes a long time to solve, you can apply formal methods of classical sensitivity analysis.

The classical methods rely on the relationships between the initial tableau and any later tableau to quickly update the optimum solution when changes are made to the coefficient of the original tableau. Finally on the state of sensitivity analysis, we are typically limited to analyzing the impact of changing only one coefficient at a time. There are few accepted techniques for changing several coefficients at once.

3.8 Change Objective Function Coefficient

A change of the coefficients of the objective function does not affect the values of the variable directly. So as we change the values of the objective function coefficients we should ensure that the optimality conditions are not violated. The range of values over which an objective function coefficient may vary without any change in the optimal solution is known as the range by those coefficient values that maintain ($cj - zj \leq 0$). The computation for the range of optimality can be categorized into two; that for the basic and also for the non-basic variables.

3.9 Changing a Right Hand Side Constraint

Right hand side constraints normally represent a limitation on the resources, and are likely to change in practice as business conditions change. An overall procedure for examining proposed changes to the right hand side of constraint is to check whether the proposed changes is within the allowable range of the right hand side of the constraint. So an optimal tableau would continue satisfying the optimal conditions regardless of the altered values of the right hand side coefficients. The change in value of the objective function per unit increase in the constraints right hand side value is known as shadow price. When Simplex method is used to solve LP problem, the values of the shadow price are found in the Z_j of the final Simplex tableau.

3.9.1 Sources and Data Collection

The data is secondary data extracted from the published annual reports and financial statements of the bank. This category of data was mainly in quantitative form. Saunders and Thornhill

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(2007) cited Stewart and Kamins (1993) that secondary data are likely to be of higher-quality than could be obtained by collecting empirical data.

A banking institution, ADB, is in the process of formulating a loan policy involving a total of GH¢ 2,119,562.00 for the year 2013. Being a full-service facility, the bank is obligated to grant loans to different clients. Table 4.1 provides the type of loans, the interest rate charged by the bank, and the probability of bad debt as estimated from past experience.

Table 4.1: Loans Available to the ADB, Sunyani.

Type of Loan	Interest Rate	Probability of Bad Debt
Commercial	0.27	0.01
Construction	0.30	0.04
Home Improvement	0.28	0.035
Mortgage	0.30	0.05
Personal (Salary)	0.28	0.015
Retail	0.30	0.03
Young Farmers (Agric)	0.25	0.075

Bad debts are assumed unrecoverable and for that matter produce no interest revenue. For policy reasons, there are limits on how ADB allocates its funds. The competition with other banking institutions in the Municipality requires that the bank:

i. Allocates at least twenty percent (20%) of the total funds to construction loan and commercial loan.
ii. To assist agriculture production in the region, agriculture loans must be at least thirty Percent (30%) of total funds.
iii. The sum of home improvement and mortgage loans should be at least fifteen percent (15%) of the total funds.
iv. Mortgage loans should be at least fifteen percent (15%) of personal, construction and agric loans.
v. Construction, mortgage and personal loans must be at least thirty percent (30%) of the total funds.
vi. The total ratio of bad debt on all loans must not exceed 0.04.

3.9.2 Proposed Loan Model for ADB, Sunyani

Base on the empirical data, the Researchers formulated the proposed model for ADB, Sunyani.

The decision variables of the model are defined as follows;

X_1 = amount for Commercial loans

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X_2 = amount for Construction loans

X_3 = amount for Home Improvement loans

X_4 = amount for Mortgage loans

X_5 = Personal loans (salary)

X_6 = Retail loans

X_7 = Young Farmers (Agric) loans

3.9.3 Objective function

The objective function of the ADB, Sunyani is to maximize its net returns, Z which comprises the difference between the revenue from interest and lost funds due to bad debts for each amount of loan disburses are shown in Table 2.

Table 2: Parameter Estimates of Profit on Loans

Loan amount	Amount of bad debts (P_iX_i)	Amount contributing to profit (1- P_i)X_i
X_1	0.010X_1	0.990X_1
X_2	0.400X_2	0.960X_2
X_3	0.035X_3	0.965X_3
X_4	0.050X_4	0.950X_4
X_5	0.015X_5	0.985X_5
X_6	0.030X_6	0.970X_6
X_7	0.075X_7	0.925X_7

Profit on loan is given by;

$$Z = [\beta_1(1-P_1)X_1 + \beta_2(1-P_2)X_2 + \beta_3(1-P_3)X_3 + \beta_4(1-P_4)X_4 + ...] - [(P_1X_1 + P_2X_2 + P_3X_3 + P_4X_4 + ...)]$$

Where $P_1 > 0$

The above can be written as;

maximize $Z = \sum_{i=1}^{n} \beta_i (1-p_i)x_i - \sum_{i=1}^{n} p_i x_i$, where

Z is the optimal solution'

βi is the coefficients of objective function, (i.e. interest rate)

Xi is the various loan items

$(1-Pi)Xi$ is the amount contributing to profit and

Pi is the probability of bad debt

Subject to: $\sum_{i=1}^{n} x_i \le w_i A_i$, where xi is an integer.

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Where $i = 1, 2, 3, \ldots, 7$ and Ai is the amount and wi is the percentage impose on the loan allocated to various loan items.

3.9.4 Implementation of the Model

Using the empirical data, the Researchers implemented the proposed model.

$$\sum_{i=1}^{7} \beta_i (1 - p_i) x_i - \sum_{i=1}^{7} p_i x_i$$

Subject to: $\sum_{i=1}^{7} x_i \leq w_i A_i$

3.9.5 Basic Assumptions of the Formulation of the above LP

A subtle assumption in the formulation above is that all loans are issued at approximately the same time. This assumption allows us to ignore the differences in the time values of the funds allocated to the different loans. All variables are restricted to nonnegative values (i.e., their numerical value will be ≥ 0). Also Non-integer values of decision variables are accepted. This is referred to as the assumption of divisibility (Amponsah, 2007)

Maximize

$$Z = 0.27(0.99X_1) + 0.3(0.96X_2) + 0.28(0.965X_3) + 0.3(0.95X_4) + 0.2875(0.985X_5) + 0.3(0.97X_6)$$
$$+0.25(0.925X_7) - 0.01X_1 - 0.04X_2 - 0.035X_3 - 0.05X_4 - 0.015X_5 - 0.03X_6 - 0.075X_7$$

this simplifies to;

$$Z = 0.2573X_1 + 0.2480X_2 + 0.2352X_3 + 0.2350X_4 + 0.2608X_5 + 0.2610X_6 + 0.1563X_7$$

The program has eight constraints.

1. Limit on total funds available (X_1, X_2, X_3, X_4, X_5, X_6, and X_7)

The total funds available isGH¢2,119,562.00

$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 \leq 2119562$

2. Construction and commercial loans

$X_1 + X_2 \geq 0.2 \times 2{,}119{,}562$

$X_1 + X_2 \geq 423912.4$

3. Limit on young farmers

$X_7 \geq 0.3 \times 2{,}119{,}562$

$X_7 \geq 635868.6$

4. Limit on home improvement and mortgage loans

$X_3 + X_4 \geq 0.15 \times 2{,}119{,}562$

$X_3 + X_4 \geq 317934.3$

5. Limit on mortgage compared to personal, commercial, and agric

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$X_4 \geq 0.15\ (X_1 + X_5 + X_7)$

$0.15\ x1 - x4 + 0.15\ x5 + 0.15\ X_7 \geq 0$

6. Limit on construction, mortgage and personal loans

$X_2 + X_4 + X_5 \geq 0.30(2119562)$

$X_2 + X_4 + X_5 \geq 635868.6$

7. Limit on bad debts

$$\frac{0.010X_1 + 0.040X_2 + 0.035X_3 + 0.059X_4 + 0.015X_5 + 0.030X_6 + 0.075X_7}{X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7} \leq 0.04$$

$$= -0.03X_1 - 0.005X_3 + 0.01X_4 - 0.025X_5 - 0.01X_6 + 0.035X_7 \leq 0$$

8. Non negativity

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0,\ X_4 \geq 0,\ X_5 \geq 0,\ X_6 \geq 0,\ X_7 \geq 0.$

That is;

Maximize $Z = 0.2573X_1 + 0.248X_2 + 0.2352X_3 + 0.235X_4 + 0.2608X_5 + 0.261X_6 + 0.1563X_7$

Subject to;

$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 \leq 2119562$

$X1 + X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 \geq 423912.4$

$0X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + X_7 \geq 635868.8$

$0X_1 + 0X_2 + X_3 + X_4 + 0X_5 + 0X_6 + 0X_7 \geq 317934.3$

$0.15X_1 + 0X_2 + 0X_3 - X_4 + 0.15X_5 + 0X_6 + 0.15X_7 \geq 0$

$-\ 0.03X_1 - 0X_2 - 0.005X_3 + 0.01X_4 - 0.01X_6 + 0.035X_7 \leq 0$

$Xi \geq 0,\ i = 1, 2, 3 \ldots 7$

3.9.6 Solution of the LP Model

The researchers used LP software to solve the linear systems as shown below;

Writing the linear system in matrix form, we have,

$Z = [0.2573 \quad 0.248 \quad 0.2352 \quad 0.235 \quad 0.2608 \quad 0.261 \quad 0.1563]$

$$P=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0.15 & 0 & 0 & -1 & 0.15 & 0 & 0.15 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ -0.03 & 0 & -0.005 & 0.01 & -0.025 & -0.01 & 0.035 \end{bmatrix} \qquad A=\begin{bmatrix} 2119562 \\ 423912.4 \\ 635868.6 \\ 317934.4 \\ 0 \\ 635868.6 \\ 0 \end{bmatrix}$$

That is, PX = A

Table 4.3: Results variables

Decision Variables	Solution	Objective Cost	Total Contribution	Reduction Cost	Dual Price
Commercial loan (X_1)	423912	0.2573	109072.5576	0	0.261
Construction loan (X_2)	0	0.2480	0	0.0091	- 0.0037
Home improvement(X_3)	317934	0.2352	74778.0768	00	- 0.1047
Mortgage loan (X_4)	0	0.2350	0	0	-0.0258
Personal loan (X_5)	635869	0.2608	165834.6352	0	0
Retail loan (X_6)	105978	0.2610	27660.258	0	-0.0002
Young farmers (X_7)	635869	0.156	99386.3247	0	0

Optimal value (Z) = 476731.8523

3.9.7 Explanation of the Solution

The results were found after nine iterations. The optimal solution or value was found to be 476732. Table 4.3 depicts variables (column 1), the optimal value of the variables (column 2), objective cost or the objective function coefficients (column 3), total contribution (column 4), reduced cost (column 4), and the dual price (column 5). These variables show that funds for the loans should be allocated for personal, retail and young farmers loans with the amounts indicated in Table 4.3.

The reduced cost for X_1, X_2, X_3, X_4, X_5, X_6 and X_7 are 0, 0.0091, 0, 0, 0, 0, and 0 respectively. The dual price for constraint (1) is 0.261, constraint (2) is – 0.0037, constraint (3) is -0.1047, constraint (4) is -0.0258, and constraint (6) is -0.0002.These are non-zero because they correspond to the active constraints at the optimum, hence their slack variables are non-basic (0), so the dual can be non-zero.

4.0 RESULTS AND DISCUSSION

The analysis revealed that, the commercial loan contributed GH¢ 109073.00, construction and mortgage loans did not contribute to the optimal solution, home improvement loan contributed

GH¢ 74778.00, personal loan contributed GH¢ 165835.00, retail loan also contributed GH¢ 27660.00 and young farmers contributed GH¢ 99386.00.

The objective function value is Z = 476732. As shown in Table 4.3, ADB, Sunyani branch should allocate GH¢ 423912.00 to commercial loans, GH¢ 317934.00 to home improvement loans, GH¢ 635869.00 to personal loans, GH¢105978.00 to retail loans and GH¢ 635869.00 to young farmers' loans and should not allocate funds to construction loans and mortgage loans since they do not contribute anything to the profit margin to ADB, Sunyani branch.

The analysis further revealed that personal loans contributed thirty four point seven nine percent (34.79%) which is the highest, followed by commercial loans which contributed twenty two point eight eight percent (22.88%), young farmers' contributed twenty point eight five percent (20.85%), home improvement loans also contributed fifteen point six nine percent (15.69%) and retail loans contributed five point eight percent (5.80%).

The results from the study showed that ADB banks in the country do not have any scientific pedagogy to give out loans. Due to this, most ADB banks are unable to optimize their profits, which intern affects their socio economic contributions in the areas in which they operate. Table 4.3 indicated that the solution value for construction and mortgage loans are all zeros which means that they do not contribute to the bank's profit in terms of loans allocated to them. The authorities of the bank should not allocate any funds to them since it will not yield any results. Commercial, home improvement, personal, retail and agric loans each respectively contributed GH¢ 109073.00, GH¢74778.00, GH¢ 165834.00, GH¢ 27660.00, and GH¢ 99386.00

It was detected that, the bank was able to achieve maximum profit due to the unit or objective cost of each decision variable. Due to this the bank should allocate funds to areas which have low probability of bad debt and interest rates which largely affect the coefficient of the objective function. The findings showed that personal loans contributed so much to the bank in terms of profit margin due to its low interest rate and low probability of bad debt. The policy makers and for that matter the loan officers at Sunyani ADB should allocate more funds to personal loans since it contributed thirty four point seven nine percent (34.79%) significantly to the bank's profit.

5.0 CONCLUSION AND RECOMMENDATIONS

Conclusions

A model which has been proposed would help ADB banks to disburse their funds available for loans more effectively, profitably and efficiently. The results from the model showed that, if ADB adopts the model they would be making an annual profit of GH¢ 476732.00 on loans alone as against GH¢ 190693.00 in 2011if they are to stick to the model. Hence the Researchers concluded that the scientific method used to develop the proposed model can have a drastic increase in the profit margin of the bank if put into use.

Recommendations

It has emerged from the conclusion that, the use of scientific methods to give out loans help banks to avoid giving out loans that do not yield any profit thereby allocating funds to areas they are sure to get good returns. Hence the Researchers recommend that ADB should adopt this model in their allocation of funds for loans. Secondly, it is recommended that managers of banks and other financial institutions be educated to use scientific methods such as the use of mathematical models to help them disburse funds of the banks and institutions more efficiently and profitably. Lastly, it is recommended that apart from loan disbursement, banks and other financial institutions should adopt the use of mathematical tools and methods in most of the businesses they conduct to improve efficiency in their work.

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